Original Article

Providing a Model for Continuous Professional Development of Mathematics Teachers Based on the Brain- Education Approach: A Systematic Review

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Abstract

Objective: Given the statistics provided by the ministry of education on the decline in the number of students interested in mathematics education and the student's weakness and disability in mathematical learning as well as the important role of mathematics in their lives and future, it is therefore necessary to take advantage of some strategies to resolve the above mentioned problems. It is undeniable that teachers play a main role in this regard. It is important to learn how to influence student's learning via continuous professional development for teachers. The purpose of the present study is to provide a model for continuous professional development of mathematics teachers based on the brain-education approach. **Method:** The present systematic review study was conducted in eight steps: subject selection, determining the exclusion and inclusion criteria, identifying search strategy using tools in databases and standard related keywords, determining location of the study area (The most important and relevant databases included in the current study were ERIC, Research gate, Science Direct, Academia.edu" and Persian database for Comprehensive Portal of Human Sciences: Institute for Humanities and Cultural Studies, SID, Magiran, Noormags, and Iranian Research Institute for Scientific Information and Documentation (IRANDOC). Studies were selected through a review of 109 abstracts and their quality was evaluated (At this stage, after designing the quality assessment tool, two researchers independently extracted and scored the references (29 related articles). Then, the data were extracted, analyzed and presented. **Findings:** To help mathematics teachers for teaching math lessons, the current study provided a model of continuous professional development for mathematics teachers based on the brain-education approach through identifying nine dimensions including brain structure and mathematical processing, application of numerical processes to mathematical achievement, extensive mathematical processing model, role of working memory and attention control in mathematics achievement, establishing a relationship between the cognitive neuroscience and mathematics training in the classroom by means of brain imaging, limitations of educational neuroscience for mathematics training, changes in teaching mathematics methods by understanding the brain function while learning mathematics and students' academic achievement in mathematics with brain-compatible learning and brain-based education package. Among them, the vast majority of studies have mostly focused on the relationship between the cognitive neuroscience and mathematics training in the classroom with the help of neuroimaging and changes in teaching mathematics methods by understanding the brain function while learning mathematics. In addition, there has been less emphasis on the dimensions of applying numerical processes to mathematics achievement and extensive mathematics processing model. **Conclusion:** The nine dimensions identified in this study can be used to provide effective mathematics teaching for students based on the brain-education approach and can be useful and effective for mathematics teachers in the context of a continuous professional development model.

Keywords: Continuous professional development for mathematics teachers, Brain education approach, Systematic review

INTRODUCTION

The movement to produce scientific documentation and literature in various fields is one of the most important events that took place in the last century and still continues. While producing and publishing an article took months or years for a long time, with the development and emergence of various communication tools, such as Internet, and access to cyberspace, today we are witnessing that we can easily achieve this goal in just a few days and the expansion of research areas of different disciplines has led to a dramatic rise in the number of scientific articles. However, it is of utmost importance to take advantage of developed methodologies and generated documentation. This matter is of paramount importance in all sciences, especially humanities and social sciences, which their research findings are closely and directly correlated to human beings; thus, in

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order to improve the quality of decision-making and policy-making, the application of research results approach has itself become a skill that policymakers in various domains need to learn and must master them well. It is important to note that different studies carried out across a wide variety of countries and centers on a specific subject matter don't have necessarily the same value; they need to be assessed and then applied through some criteria for applying the results. In this respect, evidence-based policy making is intended to evaluate the validity and reliability of various studies to set the grounds for incorporating their results in the policy making area. "Systematic review" is one of the most important methods in this regard. Reviews of existing research basically can provide valuable advantages for policymakers and practitioners to gain rich and abundant evidence and thus adopt more rigorous policies [1]. Moreover, teachers play a very important role in teaching and learning mathematics because high-quality and effective teachers are the most important factor contributing to the improvement of students' learning quality. Since learning occurs most effectively in the learning environment and the brain (hippocampus, the seat of learning and memory) is an integral area for the learning process; therefore, a brain-based approach must be incorporated into the context of the process so that the learners can achieve a desired level of performance. In addition, teachers' professional development plays a major role in enhancing students' learning and making reformations and changes in education; thus, paying special attention to their continuous professional development is essential for developing teachers' content knowledge and research. Given the above descriptions, it seems necessary to undertake a research to design a model for the continuous professional development of mathematics teachers based on the brain-training approach. As a result, given the significance of student's mathematical learning and considerable amount of research in teacher professional development, educational neuroscience and brain-based learning, it can be helpful to carry out a systematic review in this respect. The current study aimed to provide a model of continuous professional development for mathematics teachers based on the brain-training approach and in most cases, a systematic review may help provide a single and defined estimate of the desired model by making use of statistical techniques in integrating results, and thus has high power and validity for decision-making and making conclusions.

Research question

Does the continuous professional development for mathematics teachers based on the brain-education approach improve learning in secondary school students compared to other secondary school students?

Theoretical framework of research

All human behaviors and learning, such as emotion, thinking, emotion, creativity, remembrance and decision-making, are rooted deep within in the brain. In fact, the brain is the only unique and irreplaceable organ in the human body. Humans have always been concerned with understanding how the brain learns, and this has now been realized with the emergence of advanced neuroimaging technologies. The rapid growth of functional imaging techniques has provided researchers with enormous opportunities to analyze and study the brain working in healthy humans. This new knowledge will certainly promise progress for a better understanding of learning and teaching processes. The results of recent studies show that the brain undergoes changes when we learn something. Therefore, teaching is the art of changing the brain. This means that bio-neurological factors influence learning. According to the results of research, educational programs based on the brain learning have a significant impact on the students' academic achievement and optimal learning. To gain a better understanding of the brain process will increase our perception of the learning process; this in turn, leads to the improvement of educational strategies in line with how the brain learns. Therefore, understanding the neural mechanisms underlying learning and teaching processes can alter educational strategies enabling researchers to design effective and tailored educational programs for all individuals with different needs of all ages. In fact, students' brains change as a result of the experiences they gain at school. The curriculum can be defined as providing the conditions and opportunities for changing the learner's brain. Thus, the theoretical framework for curriculum development termed as brain-compatible curriculum was developed by Nouri (2011) to utilize the brain knowledge and research for improving the theatrical and practical foundations. The theory was known as Theory of Constructivist Curriculum, which consists of three main categories: underlying constructs, learning principles and curriculum strategies. The underlying category, selected as the pivotal category, is composed of two sub-categories: logical construct and theoretical construct. The logical construct explains the necessity and possibility of utilizing brain research for the curriculum, and the theoretical construct explains inference of educational strategies and guidelines based on the learning principles derived from the brain research. The learning principles group composes of three sub-categories termed as aspects of learning. The first subcategory describes "aspects of development" with its constituent concepts including: development as a constructive and dynamic process, brain flexibility, uniqueness of each learner's brain, brain as an innate meaning seeker and role of modeling in meaning making. "Emotional-physiological aspects" is the second sub-category in the learning principles with the following concepts: critical role of emotions in learning, mind-body interaction, driving role of challenge versus the deterrent role of threat and brain as a social organ. The third sub-category represents the " cognitive and meta-cognitive aspects" of learning with the following concepts: focused attention and peripheral perception, multiple levels of consciousness, brain as a parallel processor, multiple memory systems, simultaneous processing of parts and wholes, and importance of practice and training to facilitate learning and remembering. The last group, known as curriculum strategies, comprises three subcategories; each consisting of five major instructional guidelines supported at least by one learning principle. The first strategy refers to creating a "relaxed alertness" with five educational strategies: arousing emotions, creating an atmosphere free of excessive threat yet balanced challenge, adapting curriculum and training with student's developmental characteristics, activation of prior knowledge, and respect for student's individual differences. The second strategy refers to "active involvement" with the following educational strategies: Involvement of whole physiology, integrative design and organization, using multiple methods of presentation and representation and providing multi-sensory learning opportunities. Ultimately, the third strategy is termed as' reflective processing ', which includes the following five educational strategies: involvement of focused attention while at the same time understanding the peripheral environment, building the curriculum based on the student's life needs, using practice and repetition effectively, creating lifelong learning opportunities and providing the conditions to develop reflective and meta-cognitive thinking. In a nutshell, what is known as the "Theory of Neural Constructivist Curriculum " refers to a coherent and systematic set of propositions, concepts, themes and categories that describe and explain curriculum phenomena from a neural-educational perspective and considered as a plan to guide curriculum events and practical training. This theory offers a framework for designing and developing a curriculum consistent with the fundamental assumptions of constructivism philosophy. Inspired by Piaget and Vygotsky ideas, the Theory of Neural Constructivist Curriculum focuses on actively seeking and constructing meaning in the brain through interaction with the environment. This theory also puts a special emphasis on the active role of learners and at the same time the importance of social interaction in knowledge acquisition and meaning making.

Review of literature

In a study entitled as What is a brain-based learning? Asadian (2014) suggests that the formation of human brain is almost complete at birth. Both the right and left hemispheres of the brain have different functions [2]. The results of studies show that both sides of the brain handle different tasks in thinking and its function. According to the study, most people think in a particular way and process the information. Thus, a new theory known as brain-based learning was introduced into the education with specific requirements for educators and education specialists.

In a study entitled as Brain-Based Learning Studies quoted by Fogarty (2002), Mohammadi Mehr (2010) described brain-based classrooms as "brain-friendly places" [3]. They are considered as learning environments that pay attention to brain functions and their role in learning the terms of teaching and learning processes. In addition, they provide a rich emotional learning environment in which the learner is immersed in challenging experiences. Finally, according to brain-based classrooms, learners are unique and prior knowledge is used as a basis for new learning.

- In a study investigating the effect of brain-based learning instruction on the reading comprehension and learning speed of 3rd grade primary school students, Seifi [4] showed that changes in the learning environment based on the components affecting the brain (light, nutrition, oxygen, color, music and water), as well as training according to the brain-based learning principles may improve the student's reading comprehension and learning speed and has a significant impact on enhancing their learning quality. It was also found that music, as one of the effective elements of students' brain energy and motivation, strengthens learning and leads to the improvement of mathematics problem solving power.

In another study entitled as Brain-based curriculum, Talkhabi [5] reported that the curriculum goals should be connected to students' real lives. The program focuses on involving all different parts of the brain in learning process, while putting special emphasis on the role of art and music. Evaluation focuses on the learning process. Since the brain has a unique network and children develop at different rates; therefore, the curriculum development should be merely tailored to the each student's age. This type of program tends to enrich the learning environment emphasizing the appropriate emotional space. It also supports social life at school and focuses on the student's differences for learning styles and their preferences.

In a study entitled as Potential Applications of Cognitive Neuroscience to Mathematical Education, De Smedt and Grabner [6] cited some examples of articles on the potential applications of neuroscience to mathematics education (neuro- understanding, neuro-prediction, neuro-intervention) with different subject and method. We made a brief statement:

Tumpek and Obersteiner (2016) focused on measuring fraction comparison strategies with eye-tracking, suggesting that eye-tracking is a promising method for measuring strategy use in solving fraction problems.
Spuler et al. (2016) ascertained the Cognitive workload in solving mathematical problems using EEG, showing that EEG signals are able to identify an individual's workload online and have important practical implications for future applicable developments and supports the computer learning environment.

In a meta-analysis, De Smedt et al. [6] showed that there is a growing describing that the numerical skill in children's processing symbols strongly predicts future mathematical development.

In an analysis of EEG, Markely et al. (2016) supported the increasingly important role of ordinality as an important aspect for development of mathematical skills.

Poolack et al. (2016) examined how symbolic electronic are processed in mathematics to display values. Their findings promise a new way to analyze the subjective representations used in higher-order such as algebra.

In an analysis of EEG, Leikin [7] and Waismaan et al. (2016), compared adolescents with two distinct mathematics expertise and general giftedness and found that they had similar impacts on the performance, but their electrophysiological properties were significantly different, indicating their intrinsic individual's characteristics with different quality.

In an analysis of FMRI, Vogol et al. and Schilinger et al. (2016) examined the role of underlying factors in number processing, suggesting that numerical information processing may vary according to its context and this may be related to one's ability level.

In an analysis of EEG, Schilinger et al. (2016) showed that the performance pressure changes the response monitoring in a numerical task rapidly. In addition, the effectiveness of response monitoring is linearly related to the individual test anxiety focusing on the role of individual differences.

Babaei et al. (2016) utilized the findings of a brain imaging study to develop an intervention design to improve students' performance in calculating the perimeter of a shape. According to their findings, the utilization of this approach improved the students' performance. In summary, the articles in this special issue highlight that research at the intersection of cognitive neuroscience and mathematical education is still ongoing and continues to grow.

In a study entitled as Neuroscientific Studies of Mathematical Thinking and Learning: a critical look from a mathematics education perspective, Ansari and Lyons [8] and Verschaffel et al. [9] accurately described the early concerns about the integrating neuroscience research and education, which still needs further investigation. In a research entitled as Neuroscience and teaching of mathematics to improve education, Lee [10] describes the teaching of algebra in Singapore schools and the imperatives that led them to develop neuroimaging studies examining questions of curricular concerns. He focused on two issues. The first issue is related to the distinction between doing versus teaching mathematics: knowing how specific mathematical processes are implemented will not necessarily tell us how best to teach them. Second, one of the challenges in drawing useful information from the neurosciences is to bridge the divide between the laboratory and the classroom.

In a study entitled as Neuroscience studies of mathematical thinking and learning: a critical look from mathematics education viewpoint, Verschaffel et al., [9] reviewed articles on cognitive neuroscience and mathematical learning, and reminded that more necessary precautions are needed in this regard. Overall, the continuous collaboration, development between the cognitive neuroscience and education areas are of particular interest, and early concerns about the integration of neuroscience and educational research still continue.

In a study entitled as Teaching mathematics with the brain in mind: learning pure mathematics with meaning and understanding, Tanya Johnson [11] applies data and information discovered in a content analysis of research documents to create a brain-based pure math teacher resource that will help teachers teach the pure mathematics 20 programs with meaning and understanding. The lesson framework for each l considers the following brain strategies: Prior learning knowledge, attention and learning, emotion and learning, movement and learning, practice and learning, memory and learning, collaboration and learning, details and learning of each lesson include activities needed for attention, practice, definition, problem solving, project in real life situations and reflection, sharing and writing.

In a study entitled as Exploring brain-based instructional practices in secondary education classes, Constance Darcy Jack (2010) described how the brain perceives, processes, stores, and retrieves information is important to guide pedagogy, yet many schools continue to promote practices that are inconsistent with those suggested by brain research. Though brain-based teaching practices promote a more holistic approach to teaching that acknowledges the interconnectedness of the brain and how it naturally learns.

In a study entitled as Impacts of a brain-based approach on student's achievement and motivation in mathematical learning in Indonesia, Mekarina [12] suggests that the classroom action research is based on the fact that the students are less motivated to learn mathematics. One of the factors influencing learning is to provide flexibility for the students to enhance their ideal brain potentiality. The present study aimed to improve the student motivation and achievement in mathematics learning by implementing a brain-based learning approach. Brain-based learning builds a concept for learning that makes the effort to empower students' brains.

In a study entitled as The effect of brain – based learning with teacher training in division and fractions in fifth grade students of a private school, Bello [13] suggested that the use of brain-based learning principles in teaching and learning strengthened the math scores of students and improved their academic achievement.

A review of studies conducted in Iran revealed that the majorities of research on brain training have focused on one of the curriculum dimensions. A lot of focus has been placed on the learning environments and modifying it based on the brain-influencing components and evaluating students' learning quality. By analyzing neural imaging, external reviews have also sought to open a new way of analyzing the mental representations used in mathematics and focused more on individual abilities and differences. It should be noted that knowing how specific mathematical processes are represented subjectively will not necessarily improve the mathematics training and drawing useful information from the neurosciences is to bridge the divide between the laboratory and the classroom. Compared to the other studies, the present systematic review supports the idea that accessing a continuous professional development model for mathematics teachers divides the bridge between the laboratory and the classroom, because almost all research on the impact of the brain training approach have supported the positive effects of the approach on student learning. Therefore, by encouraging teachers to engage with the process and enhancing their professional abilities and competencies, we can operationalize the approach and utilize its potential to improve the student learning.

Method

This systematic review was carried out according to the Cochrane Hand book for Systematic Review. Generally, the two standard types of reviews are: 1) narrative reviews and 2) systematic reviews. Narrative reviews will be valuable when the researcher has limited access to evidence and data. However, in discussions with large volumes of data, personal opinions are not very important, and systematic review can help to accurately analyze and test the evidence [14]. In systematic review, correct, systematic and planned identification of all relevant studies can lead to more unbiased and objective reviews; if there is a discrepancy between the original studies and traditional classical reviews as well as the authors' views, systematic review can help to solve the problem.

Through finding all relevant research studies and integrating unbiased findings, systematic review summarizes evidence, updates information without the need to study all research texts, and clarifies the results through matching and comparisons techniques [15]. The results of systematic review have been echoed at national and international levels; they greatly help to find the problems and provide short-term and long-term management strategies and plans. Moreover, systematic reviews are usually carried out in different research areas once every few years and researchers around the world benefit from its results. A systematic review is a structured search based on the predefined rules and regulations. The study population was selected based on the Cochrane Hand book for Systematic Review through four phases of identification, screening, eligibility, and inclusion. In the "identification" phase, all research articles containing keywords of continuous professional development for math teachers, brain-education approach, educational neuroscience and mathematics education for searching Persian articles in Iranian databases such as Persian database for Comprehensive Portal of Human Sciences: Institute for Humanities and Cultural Studies, SID, Magiran, Noormags, and Iranian Research Institute for Scientific Information and Documentation (IRANDOC) and Ganj Iran Doc were searched. In addition, the keywords such as Continuing Professional Development for Math Teachers, Brain-based Training, Neuroscience, Math Education and their combinations were used to search for English-language articles on international databases ERIC, Research gate, ScienceDirect, Academia.edu together with Google and Google Scholar that index many sites and magazines. Reference control was also used in addition to the sites mentioned above. This search was carried out on 3.3.2018, with proper syntax. There was no time limit on searching articles. A total of 109 articles were identified. Due to the large number of articles searched, the titles of articles were initially reviewed for quality and appropriate articles, and after the removal of duplicate articles and articles with irrelevant subject matter, other reviewed articles (78 articles) were entered into the screening stage.

After studying the abstracts, articles with appropriate inclusion criteria (i.e., research published in famous scientific and research journals at home and abroad with full text referring to the continuous professional development for math teachers, brain-training approach, cognitive neuroscience and mathematics training) were screened and entered into the qualification assessment stage (49 articles). The full text of these articles was then downloaded and evaluated. At this stage, the quality assessment tool for scientific articles in humanities based on Merton's anomie theory developed by Mahram with 9 indices (title, abstract, introduction, review of literature, method (tool, population, sample, findings , conclusions, references, research report), 48 items and 4 spectra (very high, high, low, very low) was used to assess the articles. It should be noted that in order to avoid specific bias at the assessment time, the references were provided to reviewers with the author's name, journal, and other bibliographic information covered. All articles were scored after the completion of assessment. It is important to note that all the phases were performed simultaneously by two researchers and the results were compared with each other to avoid bias for selecting the references and the relevant reason was inserted in the table if rejected. If there was a discrepancy between the researchers in terms of theory, a third person will review the articles. Finally, all articles were reviewed and confirmed by one expert (29 articles).

Table 1: Researched and selected articles for inclusion analysis

Database	Database	Number of researched articles with the final syntax	Number of the final selected articles after the assessment
International	ERIC	0	26

	Research gate	13	
	ScienceDirect	11	
	Academia.edu	7	
	Google	72	
Persian-speaking language	*Institute* for *Humanities* and *Cultural Studies*	0	3
	Ganj Iran Doc	2	
	SID	0	
	Magiran	0	
	Noormags	2	
	Persian Google	2	
Total		109	29

After selecting the relevant articles through systematic review and population sampling, the full text of the articles that went through the evaluation process and qualification phase was thoroughly reviewed. Two researchers participated in the article analysis (the researchers who were engaged from the beginning of the study and had relatively good knowledge during the study), and evaluated the selected samples independently. The main purpose of this step was to extract the key findings of the papers and finally to answer the research question. Both researchers agreed on how to extract the key findings during a consultation session. The unit of analysis for article text was based on the explicit and implicit theme on the continuous professional development for the mathematics teachers based on the brain-training approach referred in the article text and its contents.

For this purpose, a checklist was developed and the information contained in various fields such as research title in Persian, research title in English, year, authors, journal / book, procedure, key findings, site address and site access time were then extracted. In order to confirm the validity of the checklist, the data were extracted from ten articles in addition to the expert's opinion involved in the research process. Having summarized the comments, the researchers modified some fields and their definitions in order to ensure that the designed fields are consistent with the articles text and the data needed to extract the text from the articles. Next, the samples were coded and categorized into larger subgroups (dimensions). The researcher's self-monitoring technique (Guba & Lincoln) was used to achieve the reliability and validity of findings. The researchers used a recursive process to extract and categorize the categories. Finally, the schema extracted from the dimensions of continuing professional development for mathematics teachers based on a brain-training approach was given to a supervising researcher, who had not studied most of the selected papers and could review them without any bias. Then a meeting was held between the researchers and the supervising researcher to finalize the dimensional classification schema and final self-assessment. PRISM Flowchart 1 illustrates the search and identification processes from inclusion to identification phases.

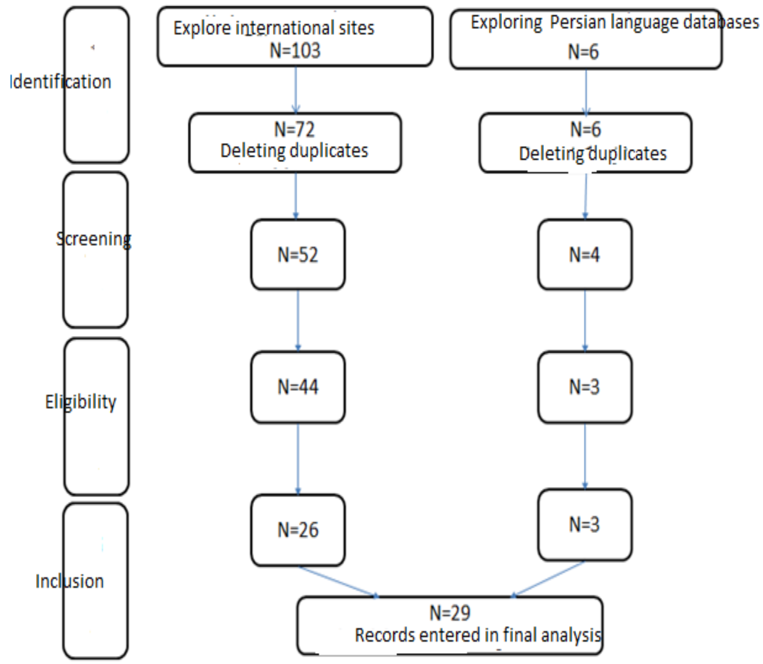

Figure 1: PRISMA flowchart: Flowchart of information searches with appropriate syntax from the search, identification and inclusion stages.

Findings

In answering the research question: Does the continuous professional development for mathematics teachers based on brain training approach improve learning in secondary school students compared to other secondary school students? Our findings showed that the identified model is based on 9 dimensions including: application of numerical processes to mathematics achievement, extensive mathematical processing model, role of working memory and attention control in mathematics achievement, establishing the relationship between cognitive neuroscience and mathematics education in the classroom by means of neuroimaging, limitations of educational neuroscience research for mathematics training, changes to teaching mathematics methods by understanding the brain function while learning mathematics, students' academic achievement in mathematics using brain-compatible learning and development of brain-based training package for helping teachers in teaching math (Table 2)

Table 2: Dimensions of continuous professional development model for mathematics teachers

Model dimensions	Number of References	References
Brain Structure and Mathematical Processing	10	[17-20] Xinlin Zhou & et al 2018, [7, 21-23] Linnea Karlsson Wirebring & et al 2015,
Application of numerical processes to mathematics achievement	7	[6,17,19,20,24-26]
Extensive mathematical processing model	6	[7,17,19,21,27, 28]
Role of working memory and attention control in mathematics achievement	10	[10,17, 18, 20, 23, 26, 27, 29,32] Linnea Karlsson Wirebring & et al 2015
Establishing the relationship between cognitive neuroscience and mathematics education in the classroom by means of neuroimaging	14	[6-10,17, 20, 21, 25,32,33] Xinlin Zhou & et al 2018, Linnea Karlsson Wirebring & et al 2015
Limitations of educational neuroscience research for mathematics training	7	[7-11, 27, 30, 29, 34]
Changes in teaching mathematical methods by understanding the brain function while learning mathematics	14	[10,11, 17,18, 20-24,27-30] Linnea Karlsson Wirebring & et al 2015,

Students' academic achievement in mathematics with brain-compatible learning	11	[7,10, 18,21,22, 28, 29, 35, 36]
	5	[13, 22,28,34,35]

As shown in Table 2, the following dimensions have received the most attention in the continuous professional development model for mathematics teachers based on the brain-training approach: establishing the relationship between the cognitive neuroscience and mathematics education in the classroom by means of neuroimaging (52% of references), changes in teaching mathematics methods by understanding the brain function while learning mathematics (52% references), students' academic achievement in mathematics with brain-compatible learning (41% references), brain structure and mathematical processing (37% of references), role of working memory and attention control in mathematics achievement (37% of references), educational neuroscience research limitations for mathematics training (26% of references), application of numerical processes to mathematics achievement (26% of references). Moreover, the dimensions which received the least attention were included: extensive mathematical processing model (22% of references) and developing brain-based training package for helping math teachers (18% of references).

Discussion

Our findings identified the dimensions of continuous professional development model for mathematics teachers based the following nine areas:

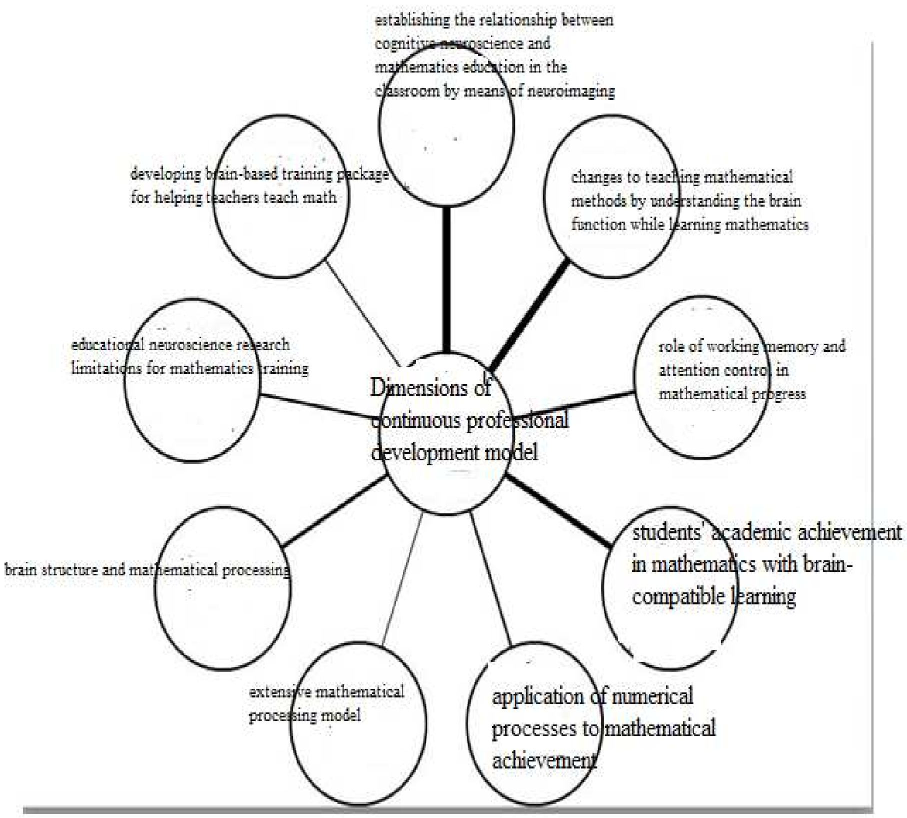

Figure 2

In Figure 2, the solid line represents the degree of focus which each dimension received in different texts: establishing the relationship between cognitive neuroscience and mathematics education in the classroom by means of neuroimaging received the most attention followed by students' academic achievement in mathematics with brain-compatible learning, brain structure and mathematical processing, role of working memory and attention control in mathematics achievement, limitations of educational neuroscience research for mathematics training and application of numerical processes to mathematics achievement. The extensive mathematical processing model and developing brain-based training package for helping teachers received the least attention.

Establishing a relationship between cognitive neuroscience and mathematics education in the classroom by means of neuroimaging

In 2010, a special issue in cognitive neuroscience and mathematical learning was published at ZDM. This article included ten papers that discussed in detail the early efforts made to learn mathematics through many interdisciplinary studies of educational neuroscience: is there any useful and potential connection between the neuroscience and education? Evidence supports the existence of a useful and credible connection. Methodologically, neuroscience offers a range of popular tools for researchers enabling them to analyze specific cognitive processes in a very rigorous way for some areas of mathematical education. De Smedt and H. Grabner have identified three types of neuroscience applications in education at the theoretical level: Neuro-understanding, neuro-prediction, neuro-intervention. Their application in mathematics education is expanding the methodology of educational researchers. Neuro-understanding: this refers to the idea that the level of neuroscience analysis can improve understanding of biological processes associated with learning school-related skills such as mathematics. Initially, this application focused on abnormalities and impairments in brain structure and function in troubled children. However, the scope of this approach has been now expanded to study individual differences in mathematics development at a much larger scale by considering individuals differences within the normal functioning range. At the other end of the continuum, there are a number of articles and emerging studies that focus on the mathematical performance of gifted and talented individuals. At the intersection between the cognitive neuroscience and mathematical education, these studies concentrated exclusively on the numerical mathematical processes. However, according to Ansari and Lyon's [8] reports, this problem has significantly penetrated to more complex mathematical learning and has gone beyond the numbers and arithmetic. In addition, the application of complex mathematical tasks in neuroscience studies caused some psychological challenges, because task complexity involves more processes that are more difficult to separate at the neurophysiological level.

- Neuro-prediction: refers to the idea that the neuroscience measures can be used to predict future individual differences (e.g., manifestation of abnormal developmental symptoms) or to respond to specific types of educational interventions. The success of this approach requires the collaboration of mathematics educators. Predicting the results in education and neuroscience would be meaningless without this cooperation. Neuro-intervention: refers to the idea that neuroscience data can be used to inform educational interventions. However, cognitive neuroscience data must be aligned with educational principles and its effects must be rigorously evaluated through educational intervention studies. As a result, mathematics education has a direct impact on brain development and gaining a full understanding of its plasticity as well as its limitations reflects an important element in the future research programs. Brain stimulation alone is not enough to induce the intervention effects [6]. Educational neuroscience is seen as an emerging discipline with its roots in cognitive neuroscience and its focus on applying the findings of neuroscience to education and posing educational questions to be pursued in neuroscientific investigation. Byrner and Fox [36] suggested that brain research findings might have useful applications in education. Since then many researchers have supported this view with several theoretical hypotheses and have attempted to link neurocognitive empirical findings with the development of educational theory and practice [7]. Neuroimaging research focuses on the underlying brain structures (the magnitude of brain activation as well as brain topographies) associated with different types of mental activities in different population groups. A variety of neuroimaging techniques (for definitions, see Grabner et al. [37]) allow researchers to obtain high-quality information on both temporal and spatial brain activity associated with different kinds of information-processing, including mathematical processing at different levels in people with varying levels of abilities. For example, the event-related brain potentials (ERP) technique offers high temporal resolution over the course of problem solving due to a precise reflection of perceptive and cognitive mechanisms. ERPs are electrophysiological measures that reflect changes in the electrical activity of the brain to external stimuli and/or cognitive processes. These measures provide information about the process in real time before the appearance of any external response [38]. Another major technique is functional magnetic resonance imaging (FMRI), which offers high spatial resolution and enables to detect differences in processing that are not evident from behavioral and ERP measures alone, thereby potentially leading to a more comprehensive understanding of the underlying processes and brain structures involved. Poolya's works (1945-1973) in mathematics education are among the most influential ones in problem solving. His four-step approach to heuristically solve problems included understanding the problem, devising a plan, carrying out the plan, and looking back. Schoenfeld [39] suggested somewhat more detailed stages of problem solving that included reading, analyzing, exploring, planning, implementing, and verifying. Poolya and Schoenfeld demonstrated that a close look into these stages can distinguish experts from non-experts in problem solving when the participants are required to cope with a non-standard problem—one for which they do not have a ready-to-use procedure. Without any connection to Poolya and Schoenfeld, Anderson et al. [40] conducted neuroimaging (FMRI) research aimed at discovering the stages of mathematical problem solving, the factors that influence the duration of these stages, and how these stages are related to the learning of a new mathematical competence. This study demonstrated that participants went through five major phases when solving a class of problems: (1) Define Phase, (2) Encode Phase, (3) Compute Phase, (4) Transform Phase and (5) Respond Phase. Two features distinguished the mastery trials during which participants came to grasp a new problem type. First, the duration of late phases of the solution process increased. Second, there was increased activation in the rostro-lateral prefrontal cortex (RLPFC) and angular gyrus (AG) regions associated with metacognition. This indicates the important contribution of reflection to successful learning. Obviously, the stages identified by Anderson et al. [40], which go beyond the task design, are in harmony with the stages devised insightfully by Poolya and Schoenfeld in their works. Anderson and colleagues provided biological validation for the big ideas of mathematics education researchers and, in this sense; theirs can be considered a neuro-validation study. At the same time, it provides us with further information about the basic cognitive abilities (visual attention, visual encoding, and motor skills), which are very often overlooked in mathematics education literature. This connection to cognitive processes can be helpful in gaining a better understanding of the effectiveness of educational practices as they are connected to specific cognitive traits. Thus, this study is also of a neuro-understanding type. One of the primary tasks of neuroscience in mathematics education is to provide a foundation for evidence-based mathematics education research. That is, growing research on mathematics education through physiological data sets, such as eye tracking, pupillary response, electroencephalography, ECG, skin reaction, respiratory rate, etc., can provide deeper and better understanding of physiological aspects of mathematics education and learning. Recording embodiments of learner's cognitive and emotional processes can help to provide important and rich insights into learners' experiences and behaviors and offer new exciting areas for research in mathematics education [23]. Despite advances in the body of knowledge related to the learning and teaching algebra, cognitive neuroscience and neuroimaging data provide new tools for an even better understanding of the processing of mathematical task. One of the major advances in this area of study occurred during the second half of the nineteenth century with the emergence of localization theories; the notion that different mental functions were related to the specific areas of the brain. The demonstration that behavioral data did not give enough information by themselves to explain mental processes led to the investigation of neural bases of behavior. As brain mapping technologies such as FMRI evolved, researchers begun to use these technologies and cognitive psychology strategies to study brain function. Mathematical educational neuroscience being considered a branch of educational neuroscience. Most recent work in cognitive neuroscience focusing on the mathematical reasoning is concerned with the processes involved in mathematics problem solving and reasoning. Menon (2010) examined various aspects of mathematical processing such as retrieval, computation, reasoning and decision- making in computational relationships and helped to find brain regions continuously involved during critical math tasks, regions with a supportive role in computing and regions that cooperate in learning computing [13]. Brain imaging techniques are able to yield information not dissolvable by more traditional, behavioral and research methods. One of the most compelling findings to emerge from this recent cognitive neuroscience research concerns the nature of algebraic processing [16]. Not only does the algebraic method of equation formulation and equation solving need a great deal more cognitive attention does the diagrammatic model method, but also that those who are not gifted mathematically but who excel in algebra achieve this excellence with a great deal of mental effort. These research findings that suggest that high cognitive effort is required in order to use algebraic methods and to excel at algebra- even for competent adults. The finding that algebraic excellence requires a great deal of mindful attention and cognitive effort should sensitize teachers and researchers to the mental demands involved in doing algebra. According to De Smedt & Grabner [6], there is great potential for linking cognitive neuroscience research and mathematics education issues. By reviewing studies entitled as Cognitive Neuroscience and Mathematical Learning, they suggested that it is very difficult to establish a relationship between the analysis levels and make conclusions from purely neuroscientific data. For example, Spuler et al (2016) reported a set of results in which different levels of mathematical tasks can be categorized based on key features of EEG time frequency data. What is difficult is to see exactly how this directly benefits educators. Perhaps, identifying which problem properties helped to group the neural features guided the authors to understand their work as something that could be transformed directly into how to teach mathematics better. In order to advance the context and increase the relevance of mathematics education issues, further empirical questions should be derived from current issues in education. For example, which is more efficient: teaching using spatial strategies in the classroom or teaching using verbal strategies? Analyzing the effects of educational interventions using neuroimaging data enables researchers to understand how the brain works and even how the brain structure changes. This approach allows researchers to better understand the neurological biological changes induced by learning and, consequently, connects these changes to the behavioral changes. Understanding which brain circuits are changed by an educational intervention can help us to better understand the mechanisms that underpin the change in the student's behavior. For example, there's already a lot of interest in the issue of which of the following interventions are more effective in math training: visual, image-based or speech-based strategies? To know which strategies are adopted by the students, we should refer to the report provided by student that can be easily diverted. However, neuroimaging can offer a better assessment of how students' strategies change. In fact, the consistency or inconsistency between the personal reports and objective brain data as well as their correlation to real mathematics learning may provide a critical perspective on how children learn new mathematical concepts. This in turn can have important implications for the ongoing debate on how to train these concepts better. Neuroimaging can provide a more objective assessment of how students' strategies may change. Furthermore, it can be used to contrast different pedagogical approaches to teach the similar skills or concepts. Using neuroimaging (in addition to behavioral measures) to contrast different types of interventions, commonalities as well as differences in their underlying mechanisms could lead to changes in students' ability and better understanding. Note that the neuroscience is essentially equated with MRI magnetic resonance imaging, as MRI-based approaches currently constitute mainstream in this field of study according to our understanding. Developmental studies are increasing our understanding of maturational changes in the human brain. In particular, structural MRI studies reveal an increase in white matter volume during childhood and adolescence suggesting an increase of connectivity in the developing brain. In addition to structural studies, functional neuroimaging provides further insight relevant to mathematics education. Moreover, functional studies can help to explain the role of specific brain regions in mathematical processing. However, more studies are needed to establish links between development of brain structures and their functional maturation. Many neuroimaging studies have focused on development of arithmetic skills in children and adults. Again, different parts of the parietal cortex, such as bilateral intra-parietal sulcus and left angular gyrus, are shown to have a crucial role in mental calculations. In contrast, other brain areas appear to mature relatively late, such as prefrontal association areas thought to be involved in mathematical cognition and other higher-order processes developing throughout childhood and adolescence. Such insight might shed some light on the transition from concrete arithmetic to the symbolic language of algebra, where students have to develop abstract reasoning skills that enable them to generalize, model, and analyze the mathematical equations and theorems. Ultimately, mathematical proficiency needs the coordinated action of many brain regions as exemplified by an influential model of algebraic equation solving [20]. In summary, we are inclined to argue that neuroscience can eventually impact on mathematics education by providing hints as to (a) what mathematics curriculum should be provided at which age, (b) which skills should be developed in parallel, and (c) how to reliably assess the effects of early diagnosis and interventions in the case of specific learning disabilities. Research on the maturation timing of brain regions involved in mathematical cognition is important as some economic models propose that earlier economic investment in education, i.e. preschool programs, always lead to larger economic return than later investments.

There is neuroscientific evidence, however, addressing the continuous development of executive functions across childhood and adolescence. Thus, educational policy-makers should be aware of the current neuroscience findings while deciding on the timing of educational investment. It is believed that neuroscience findings have not made it directly into the mathematics classroom at present. However, this should not deter research and we would like to urge investigators not only to continue but also to extend their study of educational neuroscience. Groundbreaking thoughts take time to mature and to find direct applications, and neuroscience research today is setting the scene for future developments in mathematics education.

Change in teaching mathematics through understanding brain function while learning mathematics

Donna Coch [41] believes that the majority of teacher preparation programs do not address neuroscience in their curricula. This is curious, as learning occurs in the brain in context and teachers fundamentally nurture and facilitate learning. On the one hand, merging neuroscience knowledge into teacher training programs is fraught with challenges, such as reconciling how scientific evidence is viewed and used in education, overcoming neuromyths, acknowledging the lack of direct connection between laboratory findings and classroom practices, and coordinating across different levels of analysis in neuroscience and educational practice. On the other hand, there are marked benefits to such a merger, such as deepening pedagogical content knowledge from multiple perspectives; understanding neuroplasticity and its educational implications; recognizing the power of the environment to affect neurobiology, learning, and development; and contributing to engaged, reflective practice and informed inquiry in teaching. Particularly in terms of learning equity for students and the development of a learning education culture in teacher education programs, the benefits of including neuroscience knowledge in teacher training would seem to outweigh the challenges. Brain-compatible learning and conscious learning based on the components and principles are effective and can be clarified through providing the teacher with training on the brain structure, components and principles of the brain-based learning, the importance of order in the brain structure and learning and making optimal use of teaching time requires designing dynamic lesson plan by the teacher and application of the lesson plan in the teaching process enhances the ability of the teacher to plan and relaxed alertness as components of brain-compatible learning and students employ discipline and planning in their learning process [21]. Providing the necessary brain-compatible learning principles for the teacher in order to offer conscious and intelligent training and operationalizing these trainings in the student learning process, as well as applying a variety of teaching methods to provide students with rich experiences such as using different senses in the learning process, participatory learning and the active role of learners in the learning process based on the second component of brain-compatible learning (orchestrated immersion in complex experiences) have led to improved problem-solving skills among the students. In addition, providing rich and challenging experiences in an environment with a positive emotional atmosphere will ignite curiosity and enthusiasm for learning, and is the best motive for the problem-solving process based on the intrinsic rewarding system. As a result, considering the individual differences and giving different and appropriate feedback, a learning environment based on the intrinsic rewarding system not only enhances the student's mental health and creates a calm and quiet physical environment for learning and nurturing creativity, but also brings a pleasurable experience for the students and has considerable impacts on the learning quality [21]. From the perspective of Carolyn Kieran [16], most recent research in cognitive neuroscience focusing on the mathematical reasoning are related to the processes involved in mathematical problem solving and reasoning. In one study, researchers were interested in better understanding if model and symbolic approaches rely on similar cognitive processes and analyze similar cognitive studies. Using FMRI, they found that although both methods involved in the activation of working memory and quantitative processing regions of the brain, the symbolic method triggered significantly the brain region associated with the attention needs. Increased activation of these regions allowed the researchers to deduce that generating a numerical solution to an algebraic equation has shown that "linguistic processes" play a more prominent role in the processing of symbolic stimuli. These findings led researchers to conclude that the symbolic method is more difficult than the diagram model. Brain imaging techniques are not able to generate information discoverable using traditional, behavioral and research methods. The nature of algebraic processing is one of the recent considerations in cognitive neuroscience. Not only does the algebraic method of equation formulation and equation solving require a great deal more cognitive attention does the diagrammatic model method, but also that those who are not gifted mathematically but who excel in algebra achieve this excellence by means of a great deal of mental effort. These research findings suggest that high cognitive effort is required in order to use algebraic methods and to excel at algebra- even for competent adults. Previous research has shown that students need a lot of time to be much more comfortable with algebraic symbols and to achieve mastery and power to create symbols. It has been suggested that students should begin this process at earlier age. In relation to the teaching of algebra, the model method affords children with access to algebra because it is less abstract and more visual than symbolic algebra. The findings of Kieran [16] provide new insights into the simplicity of the model method for many students; although there is no evidence that the model method relies more on visual processes than symbolic approaches. Instead, it was found that the symbolic method required more attention sources. The finding that algebraic excellence requires a great deal of mindful attention and cognitive effort should sensitize teachers and researchers to the mental demands involved in doing algebra. The subjects of the current study were students who were highly skilled in algebra. These findings should guide at least high school math teachers to stop thinking about students having trouble with algebraic elements of mathematics tasks frequently emphasized by the teachers. Teachers should change their traditional attitudes of student's algebraic activities including: This activity is simple and requires nothing but good algorithmic methods. This study can help highlight the importance of math teacher's acquaintance with findings of cognitive neuroscience research; those that could affect teachers' teaching method. By contrasting the EEG coherence in forty 8-to-9-year-old children with different math skill levels (High: High achievement, and Low achievement: LA) while performing a symbolic magnitude comparison task (i.e. determining which of two numbers is numerically larger), Gonzalez-Garrido [19] concluded that lower math achievements in children mainly associate with cognitive processing steps beyond stimulus encoding, along with the need of further attentional resources and cognitive control than their peers. Corey Drake [41] suggests that teachers need to develop their understanding of how children learn math. He offers a list of teaching methods to support student learning:

1- Posing challenging tasks that connect to children's prior understandings and out-of-school experiences,
2- Providing opportunities for children to make sense of and talk about mathematics,
3- Promoting the use of mental mathematics based on patterns in our number system.

Drake encourages teachers to make some small changes in the status quo of mathematics teaching.

1- Ask students "why" at least once every day. The best mathematics teachers will be the ones who have been prepared to empower their students as mathematicians and to teach students that mathematics makes sense.
2- Instead of looking only for whether a student's answer was right or wrong, focus on what was right in the student's work. Then build on what the student did understand in your next discussion and next task.
3- Use your textbook as a tool. Find meaningful tasks in the materials — or tasks that could be meaningful
4- Provide at least one opportunity each day for students to solve and explain problems mentally (without pencils, paper, calculators, or computers).
5- Give students a chance to discover and make sense math.
6- Teachers need to devote a lot of time to hear and respond to children's ideas.

He believes that classroom-level changes will ultimately only lead to, at best, incremental change in the status quo of the larger systems of mathematics education. However, expecting teachers to have the sole burden for changing these systems is not only ineffective, but also ethically problematic. Drake recommends mathematics teachers to teach math based on the brain training approach principles.

In addition, Boaler and Chen [42] suggest that math should be taught more visually. New evidence on how the brain functions when we think about mathematics could change the way mathematics is taught.

- Training people on ways to perceive and represent fingers results in higher math achievement,".
- Schools do not know about this important brain research and many schools even ban students from using fingers in classrooms. While new research suggests that stopping students from counting on their fingers is akin to halting their mathematical development.

Because the research shows that everyone uses visual pathways when they work on mathematics, parents and teachers need to develop the visual areas of children's brains. They can do this by:

1. Using visuals, manipulatives and motion in mathematics teaching and parentin
2. Providing opportunities for students to use drawing, visualizing or working with models in mathematics
3. Teaching algebra visually through pattern study and generalization
4. Asking students, at regular intervals, how they see mathematical ideas
5. Asking students to represent mathematical ideas in a multitude of ways, such as through pictures, models, graphs, even doodles or cartoons

According to the authors, there is an urgent need to change the ways mathematics is offered to learners in order for them to function well in modern society as almost all new jobs require employees to make sense of "big data," which includes seeing data patterns visually. Zadina [28] recalls that educators must stay up to date on what we are learning about the brain, emotions, motivation, and physiology from scientific research. This information can inform our actual practices in the classroom in the form of our understanding of students and our ability to design and implement more effective lessons. They should be presented by properly trained presenters; we can make this leap from research to professional development. When teachers understand more about developmental stages and learning differences illuminated through neuroimaging studies, this can affect their attitudes and practices and potentially lead to better outcomes for students. Deeper and broader implications can be derived from substantial bodies of literature on processes underlying thinking and learning that are invisible to classroom teachers. Understanding these processes can help educators explore alternative or targeted interventions. The teachers who are unable to understand these processes might just "drill and kill," hoping that enough repetition and practice would break through the barrier. The teachers who understand the process know that math problems are actually working memory problems. Some students quickly forget verbal information or can't hold information long enough in working memory to complete a math problem. A knowledgeable teacher firstly improves working memory capacity through providing more teaching opportunities. Fluid intelligence can be improved by rehearsing a working memory task. Research using functional magnetic resonance imaging (FMRI) shows more demand on working memory when students are initially learning and the cognitive load is higher than later in the learning process. Teachers who are able to understand cognitive load theory can teach differently, allocate more time early in the process than later and design lessons in ways that address the effects of cognitive load and working memory limitations. Working memory capacity was predicted to improve by the ability to control attention. The value of classroom strategies lies in increasing attention. Investing in preschool attention training gives a better return on investment dollars than remediation later in the education process. Students often have serious anxiety or stress-related issues. An understanding of the biology of threat, of how anxiety and stress impact learning, and the nature of our traumatized students can help educators understand why a method can actually inhibit learning. Neuroimaging studies have revealed the complexity of interactions of brain regions and revealed multiple pathways involved in learning. An Educational Neuroscientist can guide teachers toward designing lessons with multiple pathways to understand and practice offering options for different and struggling learners. Learning styles theory draws a distinction between learning visually, auditorially, or kinesthetically. Neuroscience research highlights the importance of vision in learning. In continuous professional development, math teachers should be trained to make use of more visual lessons. Hasani et al. [23] suggested that many different learning theories have been developed until now, each focusing on mental and behavioral processes aimed at enabling individuals and using brain capacities for learning and retrieval. It is a scientific fact that knowing how the brain works and the mental and cognitive processes of audience can assert the claim that the math teacher can help others use their own brain capabilities and functions in an effective way in order to achieve a meaningful understanding of mathematical topics. Learning is the best thing the human brain can do. Scientists believe that the brain makes meaning and is a crucial element for learning. Mathematics education is a branch of the humanities that has gained an important position in the world's scientific circles particularly in developed countries in recent years. Therefore, scientific identification of learners' problems in mathematics and planning and making an effort to solve them by teachers, planners and authors based on brain-based learning principles lead to academic achievement in mathematics including qualitative and quantitative changes in developing competent behavior of math students. In mathematics education, the application of "brain-learning" approach entails paying special attention to structures, and a critical issue in mathematics education is the mutual adaptation to the environment, experiencing information processing and natural logical constructs related to brain with mathematics, as well as training programs and methods. The integrative education model by Clark based on the Jung theory of brain-mind development including thoughts, feelings, senses, and intuitions functions focused on the brain-training approach. This model organizes the curriculum through four functions of 1. Thinking 2. Intuition, 3. Sense and 4. Emotion. According to Piaget et al., the inability to cope with math is due to the excessively rapid passage from the qualitative to the quantitative. Therefore, the reviewed literature on human learning with a brain-based approach states that the use of mediatory teaching practices are effective on the processing of mathematical content and cognitive success is of great importance in education. In information processing, new teaching methods in family and information processing patterns that aim to improve and reinforce mental abilities can be utilized aligned with mathematics lessons; an issue that has been addressed in theoretical theories suggested by Piaget, Azubel, Lucas, and Bruner. Controlled and deliberate emotional experiences, which have been considered as classroom intrusive factors by some teachers, can have some implications for empowering individuals to improve students' learning and mental ability and enhance their awareness and learning perception. Given the global trends towards collaborative research between neuroscience and education, teachers, as learning specialist, need to be aware of how the brain learns. Teacher knowledge of brain functions and proper use of brain-based learning principles during teaching as well as offering challenging teaching methods in mathematics lessons will greatly contribute to mathematical academic achievement and positive attitudes and tendencies. Enriched learning environment (rich learning environment, emotional regulation, and information processing) brings additional benefits to the learners resulting in the positive effects on the student's mathematical attitudes and achievement, so that the low mathematical anxiety leads to less abstract conceptualization and helps to further consolidate the position of teaching strategies in mathematics teaching. Effectiveness of the emotional and attitude aspects on the mathematics education is a serious and undeniable issue that enhances one's ability to learn mathematics and performance to perceive mathematical concepts. Research findings indicate that attitude is among the factors contributing to the academic achievement and enhances the knowledge structure and information processing process of the student. As a result, the attitude promotes and stimulates brain activities such as thinking and learning; therefore, in order to create deep and long-lasting learning especially in mathematics education, educators must first develop positive attitudes and tendencies towards the lesson and math class. According to the findings of Jensen's new research, the threatened environment may even cause chemical imbalances. Serotonin is the ultimate crucial modulator of emotional regulation and behaviors. Violence and aggression increase as serotonin levels decline. This imbalance leads to aggressive behavior, and both relaxation and threat continuously overshadow modeling and complex problem-solving. Linna Karlsson Wirebring et al. (2015) suggest that a dominant mathematics teaching method is to present a solution method and let pupils repeatedly practice it. An alternative method is to let pupils create a solution method themselves. They present two approaches. The first focuses on presenting some tasks and then offers some suggestions for solution method. Such teaching methods are guaranteed to lead to learning in the short term but conceptually; they appear to have much in common with 'rote learning': the process of learning something by repeating it until you remember it rather than by understanding the meaning of it. However, in spite of being short-term efficient, there is data indicating that teaching based only on such methods fails to enhance students' long-term development of conceptual understanding. They introduced Creative Mathematically Founded reasoning, suggesting that encouraging the individuals to create a solution method themselves should be superior for promoting mathematical learning. Their findings showed that creative mathematically founded reasoning (CMR) will promote better performance attest one week after training than algorithmic reasoning. Acquainting math teachers with these methods can guarantee improved student performance in mathematics.

As a high school teacher, Connie White says that his passion is to help teachers incorporate strategies, methods and technology tools that foster student engagement and learning. Further, he helps students understand how they learn and empower them to take control over their learning. He has found that research about the brain can catalyze our understanding of how students learn and how teachers, in response, should mobilize.

1. Encouragement makes a difference

First and foremost, teachers have a tremendous impact on student learning. If the student thinks the teacher likes or cares about him and the student thinks the teacher believes in him and his abilities, then achievement improves. Conversely, if a teacher expresses a lack of confidence in the ability of the student, learning declines dramatically and failure becomes a self-fulfilling prophecy. There is science behind it: When students feel positive about their learning environment, endorphins are released in the brain. Endorphins produce a feeling of euphoria and stimulate the frontal lobes, thereby making the learning experience more pleasurable and successful. It is critical that teachers provide a positive, safe and nurturing environment.

2. Brain plasticity: "use it or lose it"

Research has also found that the brain is a "use it or lose it" organ. This is good news for learners of all ages: With activity and use, neural circuits grow and re-wire. Neuro-pathways and connections become stronger and information is stored and retained. It can also be more readily retrieved. Neuroscientists call this process brain plasticity. If students constantly memorize facts, then their rote memory pathways will get stronger. If we provide opportunities for students to think, analyze and solve problem, students become more adept at critical thinking.

3. Mix it up: multifaceted instruction

Learning is also increased and retrieval is made easier when different types of memory pathways are incorporated when teaching. The brain needs multifaceted experiences such as multi-sensory input, scaffolding on previous learning, stories and reciprocal teaching. Technology can be incorporated in a multitude of ways by teachers and students. In addition to auditory and visual strategies, kinesthetic pathways should be activated as well.

4. Make it relevant: project-based learning

Another key component to teaching and learning is relevancy. How many times have you heard it said (or said yourself), When will I ever need to know this in the future? If the learning experience is designed to solve a real-world, meaningful problem, the information is more likely to make it to long term memory. In project-based learning, students reinforce executive function skills such as goal setting, managing a timeline, brainstorming solutions, collaborating, revising and presenting to a public audience. Creating an innovative final project using technology to demonstrate learning can be accomplished in a variety of ways.

5. Use time effectively, and remember to reflect

Adding a component of reflection to the learning experience will yield tremendous gains as well. When students are given an opportunity to simply think about, or reflect on, what they have just taken in, learning is deepened and memory consolidation improved. The average attention span of a child is 10 to 20 minutes, so any learning experience should change person, place or topic according to that timeframe.

As a final note, positive environment, brain plasticity, multi-faceted instruction, real world connections, reflection and attention span are only a few brain-based factors that teachers can merge into their lesson design. Intentional planning on the front end can make a tremendous difference within the classroom.

David A. Sousa [32] refers the significance of using neuroscience research results to help educational advisers to make educational decisions. Recently, brain-imaging technology has brought the field of neuroscience into the study of teaching and learning mathematics. Imaging technologies have allowed scientists to determine which areas of the brain are active when the mind engages in mathematics. In fact, this technology has given researchers and educators a new piece of the learning puzzle. It is now possible to compare learning theories in mathematics to neurological analyses of how the brain physically functions while it is doing mathematics. He reviews knowledge of the human brain's evolution and physiology, as well as current theories about teaching and learning and merges that knowledge with new information from brain imaging. Susan gives a detailed explanation of memory and brain imaging-based learning and organizes the chapters according to three age bands; kindergarten, preadolescence and adolescence in order to highlight the differences in the developing brain and the impact of those differences on students' ability to learn mathematics. He skillfully layers information from brain scans to show the parts of the brain that are active during different mathematics activities. In addition, he advocates using mathematical reasoning and meaning based activities like the division of fractions and discourages the use of tricks in teaching. Meaningful learning of the topic like fractions may help students connect the division of fractions to larger mathematical ideas. Sousa also advocates that teacher's planning with knowledge of working memory (understanding that students can only hold about five or six new pieces of information in their working memory), limiting the number of objectives per lesson and identifying and resolving mathematical difficulties resulting from environmental factors are among the factors which have a direct impact on teaching based on the teacher's awareness of how the brain functions. Environmental and instructional settings that make a child feel anxious may also contribute to mathematics difficulties. Sousa documents some of the physiological effects of stress and anxiety on memory and cognitive function and offers suggestions for teachers who are interested in developing mathematics learning environments that mitigate anxiety. Sousa argues that a teacher's perceptions about how children should be taught and assessed can influence how a disability is perceived or diagnosed. A child who struggles with rote memorization might be diagnosed as learning disabled by an instructor who relies heavily on memory-based instruction. That same child might have strengths in problem solving and would not be diagnosed as learning disabled by a teacher who attends to problem solving over rote memorization. Johnson [11] suggests that students continue to feel inadequate in pure math classes and algebra lessons. Changes needed to resolve this disturbing situation include teachers themselves altering their teaching strategies to help minimize the existing problems in the pure math program. Research on the multiple intelligence theory reminds us of the different student learning styles and the fact that, more than one type of teaching strategy should be used to deliver the pure math program. He notes that her current research on the science of learning has brought to light some very interesting ideas of how a student's brain works and the applications of this work to classroom practice. As teachers, we can translate this information into classroom practice in order to help our students learn pure mathematics with meaning and understanding.

John Munro [26] investigates mathematics teaching methods with regard to working memory problems:

A). An approach to teaching any mathematics idea that includes teacher scaffolding of working memory processes while teaching the idea. For example, teaching students to add two fractions with different denominators is explained:

1. Encode the task in working memory; scaffold students to interpret the task: Read the task and say what it says. Make a picture of what it says.
2. Stimulate what students already know about this type of task; this provides the existing knowledge base for encoding and representing the new ideas; scaffold students to say what they know about the type of task: What types of fractions can you add? Can you add 2/6+ 3/6 =? Write down some other tasks you can add.
3. Guide students to link the new task with ones they already can do; this assists them to focus on the particular features of the new task; scaffold students to say: How do these tasks differ from the one we are working on?
4. Guide students to encode the type of problem in working memory for later storage in long term memory; scaffold students to ask: Can I make task I can't do like the ones I can do?
5. Stimulate what students already know about key components of the new task; this assists them to retrieve additional relevant aspects of their existing knowledge; scaffold students to recall how they can say each fraction

in other ways: What are other fractions that say the same as 1/3? Write down some other tasks you can add. Repeat for ½. What do these two sets of pictures show? How could you use what they tell us?

6. Stimulate students to use the new links to complete a specific task; this assists them to encode a particular example in context in working memory; Scaffold students to see how they can use the alternative names for 1/2 and 1/3 to solve the task 1/2 + 1/3 =. Ask: Remember we want to find two fractions that are the same as 1/2 and 1/3. Can you find another fraction for 1/2 and 1/3 that have the same denominator? Cue the students to rehearse the new links
7. Repeat with similar particular tasks; this guides students to encode the type of task in working memory; Scaffold students to use the procedure with similar tasks using the worked example as a model, for example, 2/3 + 1/4 =. Cue them to say what they will do before they begin using the earlier task. Scaffold each step. Have them work through a set of practice tasks. Before they begin each task, ask them to say what they will do.
8. Guide the students to identify and describe the new procedure and to practise applying it; this guides students to encode the new procedure in working memory; cue the students to review the tasks they have completed and identify the procedure. Guide them to recognize key aspects of the procedure.
9. Guide the students to identify when to use the new procedure this guides students to encode in working memory the types of contexts in which they will use the new procedure; cue the students to note how these tasks differ from the ones they could already do. Guide them to give a name to the new type of task so that it is distinguished from the earlier type. Ask them to make up tasks that are not ready and ones that are ready. Classify tasks. Say what they know about the two types of tasks.
10. Guide the students to automatize what they know about how to add two fractions. Cue students to decide rapidly whether an addition of fractions is ready to add and if so to work out the new denominator, numerators and add them.

B). There are various working memory strategies that the teacher can teach students to use them whenever they are engaged in mathematics. The following teaching methods can be used to encode and manipulate knowledge in working memory:

1. Teacher should stimulate explicitly what they already know about the task they will learn. Your teaching can include the relevant mathematics procedures, concepts, the mathematics symbolism and the factual knowledge they will need to use. Students can learn to link the mathematics procedures you will teach with the procedures they already know.
2. Teacher should teach the students to say and paraphrase relevant mathematics information such as tasks, number sentences and mathematics ideas. This helps them to ‘read’ the number sentences into working memory. Have them ‘tell themselves’ what concrete patterns, pictures of mathematics ideas and mathematics actions show.
3. Teacher should teach the students to visualize mathematics ideas, for example, number sentences. Scaffold them to use these strategies and give them time to do so.
4. Teacher should teach the students to learn new ideas first through 2 or 3 specific examples and then to extract the procedure. Ask them to talk about what the three examples show.
5. Teacher should teach the students to learn each mental action, for example, adding or subtracting as physical actions first and gradually internalize them.
6. Teacher should use a sequence of self-instructional strategies to guide their way through any task. When they are doing mathematics tasks, teach them to

Say what a task says;
Visualize it;
Say what the solution will be like;
Categorizes the task; say what they will do first, second, …;
Plan what they will do the task.

7. Teacher should organize new mathematics ideas into categories. When they have learnt a new mathematics idea, have them recognize instances of it, teach it as a category and teach them a name for the category.
8. Teacher should review regularly how they learn the ideas and the thinking strategies they used. This helps them learn a repertoire of working memory rehearsal and transformational strategies that they can use on later occasions.
9. Teacher should teach the students to say what they have learnt when they have learnt a new mathematics idea. Ask them to say how the new idea is similar to and different from what they already knew. Teach them explicitly to link it with what they knew. This helps them to store the new mathematics knowledge in memory.
10. Teacher should teach the students to automatize the new knowledge by teaching independent use of the idea initially. Scaffold them to do more of the idea and gradually remove the scaffolding as they do more of the idea by themselves. Have them practise doing it.

In summary, understanding the brain's function while teaching mathematics leads to significant changes in teaching methods, which will probably help students to learn mathematics better and easier.

Brain Structure and Mathematical Processing

Jie Liu et al [18] suggested that although numerous studies have shown that brain regions around the intraparietal sulcus (IPS) play an important role in general mathematical or numerical processing, little is known about the specific neural correlates for processing mathematical principles. Therefore, they compared the activation intensity, multi-voxel activation patterns, and functional connectivity (FC) related to processing mathematical principles (including arithmetic and logic) with those related to arithmetic. Twenty right-handed undergraduates (10 male; aged 18-25 years) participated in their study. Results of whole-brain univariate analysis showed that brain activity in the left angular gyrus (AG) was consistently stronger for mathematical principles than for computation. Multiple-voxel activation patterns at the left middle temporal gyrus (MTG) differed between mathematical principles and arithmetical computation. Additionally, psychophysiological interaction analysis showed that the functional connectivities between (1) the left middle temporal gyrus and the intraparietal sulcus, (2) left middle temporal gyrus and left inferior frontal cortex (IFG), and (3) the intraparietal sulcus (IPS) and left angular gyrus were consistently stronger for mathematical principles than for computation. As the AG, MTG and orbital part of IFG were key regions of the semantic system, these results provided direct evidence that the semantic system plays an important role in the processing of mathematical principles. This study determined how processing mathematical principles differs from mathematical computation in the brain in terms of activity levels and functional connections. Results from the univariate, multi-voxel, and functional connectivity analyses consistently revealed that the left angular gyrus, left middle temporal gyrus, and left inferior frontal gyrus were more involved in the processing of mathematical principles than in computation. These regions are connected with the intraparietal sulcus, the core region involved in mathematical processing.

As the AG, MTG and orbital part of IFG were key regions of the semantic system, these results provide direct evidence for a crucial role of the semantic system in the processing of mathematical principles. Kieran [12] suggests that most recent work in cognitive neuroscience focusing on the mathematical reasoning is concerned with the processes involved in mathematical problem solving and reasoning. Menon (2010) examined various aspects of mathematical processing such as retrieval, computation, reasoning and decision making in computational relationships and helped to identify brain regions continuously involved during critical math tasks, regions with a supportive role in computing and regions that cooperate in learning computing. Gonzalez-Garrido [19] report that EEG coherence represents a useful measure of brain functional connectivity. They aimed to contrast the EEG coherence in forty 8-to-9-year-old children with different math skill levels (HA, High achievement and LA: Low achievement) according to their arithmetic scores in the Fourth Edition of the Wide Range Achievement Test (WRAT-4) while performing a symbolic magnitude comparison task (i.e. determining which of two numbers is numerically larger). The analysis showed significantly greater coherence over the right hemisphere in the two groups, but with a distinctive connectivity pattern. Whereas functional connectivity in the HA group was predominant in parietal areas, especially involving beta frequencies, the LA group showed more extensive front parietal relationships, with higher participation of delta, theta and alpha band frequencies, along with a distinct time–frequency domain expression. Ana Susac and Sven Braeutigam [20] believe that functional studies can help to elucidate the role of specific brain regions in mathematical processing. However, additional studies are needed to establish links between development of brain structures and their functional maturation. Many neuroimaging studies have focused on development of arithmetic skills in children and adults. Again, different parts of the parietal cortex, such as bilateral intra-parietal sulcus and left angular gyrus, are shown to have a crucial role in mental calculations. In contrast, other brain areas appear to mature relatively late, such as prefrontal association areas thought to be involved in mathematical cognition and other higher-order processes developing throughout childhood and adolescence. Roza Leikin [7] points out that individual differences focus on structural features and brain function and neuroimaging studies demonstrate the neural correlates of mathematical difficulties and disabilities. He elaborates the formulation of the neural efficiency hypothesis of intelligence, which states that brighter individuals display lower (more efficient) brain activation while performing cognitive tasks. Neural efficiency is related to individuals' expertise in a given field. When it comes to performing difficult and challenging tasks, more intelligent individuals exhibit higher brain activity. Linnea Karlsson Wirebring et al. (2015) compared two methods of creative mathematical founded reasoning (CMR) and algorithmic reasoning (AR), showing that CMR leads to less involvement in left angular gyrus compared to AR. Creative mathematically founded reasoning leads to better performance and relatively lower angular gyrus brain activity in the long-term compared to algorithmic reasoning. In addition, these results demonstrate that the right superior parietal cortex is pivotal for mathematical performance in general, possibly reflecting attentional and/or working memory contributions to complex mathematics.

Role of working memory and attention control in mathematics achievement

Azalia Herma [17] explains that simple cognitive process such as emotions processing, sense of love-not-love, and logic occur in the amygdala, the motion system, and the neocortex. More complex cognitive understanding such as visualization, creativity, focus, and intelligence occurs in the prefrontal cortex. Most people use simple cognitive understanding of the area: 1) Orbitofrontal prefrontal cortex, which is responsible for emotions and other controls in the various regions of the brain, the most active working mainly for balancing process simple cognitive understanding. 2) Dorsolateral prefrontal cortex, which is responsible for working memory and mental manipulation, more dormant. The prefrontal cortex is divided into two parts, which can only be activated one at a time or orbitofrontal or dorsolateral cortex. Since most people use simple cognitive understanding of the area, so orbitofrontal cortex is more active and dorsolateral cortex is dormant. This limitation is causing the human brain cannot maximize employment; causing remarkable ability of human beings can only be traced surface. It can be concluded that creating mathematical intelligence is not merely stimulate the right brain and left brain, but it also reduces the use of simple cognitive understanding of the process and create a special trick to activate the prefrontal cortex. The amygdala also plays a role in the consolidation of memory, attention, perception, and reactions associated with emotions. Amygdala is also one area which enables the consolidation of long-term memory, especially if it involves emotional memories. Amygdala allows humans to adapt and store information more efficiently. An event that involves emotion strong enough to trigger the secretion of epinephrine and glucocorticoids by the adrenal glands. These hormones then trigger the secretion of norepinephrine into the amygdala, induces the amygdala work to consolidate the memory of the incident. Anxiety or certain antipathy experienced by students when faced with a problem stimulates amygdala and enhances anxiety and fear. This may happen when students feel depressed over the material being taught, teaching staff are incompetent, do not support the learning environment, as well as other personal reasons such as lack of confidence, inaccurate and lazy. In addition, equalization functions of the left brain and right brain improves performance on a math problem and is a strong positive stimulus to the amygdala that can be embedded into one's memory without creating resentment, fear or anxiety.

According to Carolyn Kieran [16], in one study, researchers were interested in better understanding whether model and symbolic approaches rely on similar cognitive processes and analyze similar cognitive studies. Using FMRI, they found that although both methods were associated with activation of working memory and quantitative processing regions of the brain, the symbolic method greatly activates the brain region associated with attention needs. The additional activation of these regions allowed the researchers to deduce that generating a numerical solution to an algebraic equation requires extra attention and executive resources compared to diagram model. According to De Smedt and Verschaffel [30], recent research in cognitive neuroscience suggests that some solution methods have a higher cognitive demand than others. These data suggest that these methods may not be appropriate for training at a young age, when working memory and attention control are not yet complete. Gonzalez-Garrido [19] states that EEG beta fluctuations are associated with working memory processing and attention depending on the mathematical achievements. They need further attentional resources and cognitive control than their peers. Zadina [28] suggests that some students quickly forget verbal information or can’t hold information long enough in working memory to complete a math problem. A knowledgeable teacher firstly improves working memory capacity would be more effective than having students only practicing the mathematical problems. Fluid intelligence can be improved by rehearsing a working memory task. Research using functional magnetic resonance imaging (FMRI) shows more demand on working memory when students are initially learning, when the cognitive load is higher than later in the learning process.

In one study, Linnea Karlsson Wirebring et al. (2015) compared the impact of two teaching methods AR and CMR using Functional Magnetic Resonance Imaging (FMRI) on the mathematical function as well as brain activity and concluded that working memory and sharpness ANS (numerical approximation system) will be significant predictors of individual differences in mathematical performance (independent of the teaching method). J. Landeira-Fernandez [31] believes that the working memory model provides a useful framework for understanding the role of the different cognitive mechanisms involved in these mathematical skills. He reviewed several neuropsychological and neuroimaging studies, suggesting that mathematical performance depends on working memory resources. The paper begins with a description of the different working memory components. He then presents evidences that suggest that each working memory component plays a crucial role in mathematical problem solving. He also reviews numerous studies that show that working memory and mathematical thinking share a considerable number of neural circuitries within the posterior parietal cortex and prefrontal regions. Finally, he discusses how anxiety might jeopardize working memory capacity and thus reduce performance in solving mathematical problems. Measures that increase working memory capacity might improve mathematical problem-solving achievement. Interventions based on controlling the negative feelings that precede mathematical performance might also be helpful for people who suffer from mathematical anxiety. Mathematical thinking is an important mental function for everyday life. It requires highly diverse cognitive abilities, ranging from decomposing and understanding the mathematical problem verbally to abstract symbol manipulation in a visual-spatial representation. For example, learning to read can occur independently of any mathematical knowledge. Nonetheless, reading is a necessary condition to solve mathematical problems. Therefore, different cognitive domains that store and manipulate verbal and visual-spatial information while solving a mathematical problem represent crucial factors for successful performance on these tasks.

A considerable amount of empirical evidence from the past 30 years clearly indicates that different working memory components are critically involved in the solution of mathematical problems that require more than just memory retrieval. Indeed, working memory is increasingly more involved in mathematical reasoning as more intermediate steps are required to solve a mathematical problem. Neuroimaging studies corroborate this premise. Although the precise neural circuitry that underlies mathematical thinking is not completely clear, several studies have reported that working memory and mathematical tasks likely recruit the same brain structures within the posterior parietal and prefrontal cortices. Understanding the neuropsychological mechanism and neural circuitries of mathematical thinking might help develop measures that can improve mathematical problem-solving achievement.

The fact that adequate mathematical performance requires an optimal working memory capacity suggests that improving working memory resources might enhance mathematical performance. For example, Witt (2011) recently reported that children who received working memory training exhibited a significant improvement in mathematical problem solving compared with matched subjected who did not receive working memory training. In a study aimed at identifying the origin of quantitative competencies in the first grade, Geary [25] predicts the starting point for mathematical achievements and its development in fifth grade. By presenting multilevel models, he demonstrated that intelligence, processing speed and the central executive component of working memory predicted achievement or achievement growth in mathematics as a contrast domain of word reading. The phonological loop was uniquely predictive of word reading and the visuospatial sketch pad of mathematics. Early fluency in the processing and manipulating numerical set size and Arabic numerals, accurate use of sophisticated counting procedures for solving addition and accuracy in making placements on a mathematical number line were uniquely predictive of mathematics achievement. Use of memory-based processes to solve addition problems predicted mathematics and reading achievement but in different ways. The results show that the early quantitative competencies uniquely contribute to mathematical learning.

Munro [26] examines the role of working memory in typical mathematics tasks, procedures for diagnosing working memory influences on mathematics learning difficulties and intervention strategies for enhancing working memory processes during mathematics learning. To explain the role of working memory, he proposes multi-component model of Baddeley working memory. It seems logical that success in mathematics learning necessitates the efficient use of student's working memory.

However, Raghubar [43] notes the complexity of this relationship and the likelihood that for any individual it will depend on a wide range of factors that influence how the individual interacts with the mathematics information (either the teaching information or the information specifying a problem or task). These include personal factors such as age and skill level, mathematics content factors and characteristics of learning –teaching context such as level of mastery being targeted, language of instruction and the formats in which the mathematics information is presented. He notes the need for 'a sufficiently comprehensive model of mathematical processing, particularly in relation to skill acquisition that can handle current findings on working memory as well as provide the basis from which to guide new discoveries and inform practice.

Children with math difficulties differ from their peers without difficulties in each aspect of their working memory processes; in verbal working memory, in static and/or dynamic visual–spatial memory processing, in numerical working memory and in backward digit span tasks. Given a lack of consistency across studies about how to measure the components of verbal and visual–spatial working memory, you can see various trends across the age span of school, for example,

1. Executive and visual–spatial memory processes are used more during learning new mathematical skills/concepts and the phonological loop processes after a skill has been learned.
2. Longitudinal studies suggest that some executive processes may be more generic in terms of supporting learning, while others, such as visual–spatial working memory may be more specific to early mathematical learning and verbal processes become more prominent at older ages
3. Different aspects of working memory indirectly mediate different aspects of mathematical performance for dyscalculic children.
4. Working memory is linked with other factors in mathematics learning such as students' ability to use and focus their 'learning attention'.

Abedi et al. [44], Khodaie et al. [45], Najafifard et al. [46], Orkie [47], and Banie Jamali [43] all concluded that paying attention to working memory training underlies mathematical learning. Therefore, this issue is of significant importance in the continuous professional development model for mathematics teachers. It is important to note that teachers and schools can make a valuable contribution to such a model as teachers observe how their students respond to their mathematics information. They at least develop a visual awareness on the impact of the working memory process on students' mathematics understanding so that most researchers from other disciplines reject it. Providing insights into how students respond to classroom training yields potentially useful information.

Limitations of educational neuroscience research for mathematics education

From a critical perspective of mathematics education, many studies investigated the relationship between the cognitive neuroscience and mathematics. Verschaffel, Lehtinen, and Van Dooren [9] have pointed out precautionary considerations that should be taken into account:

1. There is a strong tension between the practical and technical limitations of existing neuroscience approaches and the need for ecological access.
2. Adult populations are experimental participants in neuroscience studies.
3. Tasks used in these studies are often very elementary and differ from those typically solved in class.
4. Most neuroscience studies in mathematics education have examined the mathematics function in a relative isolation.
5. The mathematics development cannot be considered separately from the learning and teaching fields.
6. Participants' learning history as well as their learning environment is typically considered to be a confounding

variable reported in most studies. While these variables are crucial, their variations may exert wide ranging effects on brain structure and activity.

7. Not knowing how mathematics is taught in school, cognitive neuroscientists are at risk for simple new experiments that have little or no relevance to educational activities.

Ansari and Lyons [8] discuss key issues related to cognitive neuroscience and mathematics education. One might be related to the selection of adults as participants in these studies. They argue that the adult analysis can only familiar us with some cognitive techniques used in neurosciences for mathematics learning. In particular, there is a need to analyze children of different ages and backgrounds while acquiring specific math skills. They also noted that there is a need for improved ecological validity of experimental and specific conditions to measure mathematical processing. In particular, they argued that a lot of studies used empirically controlled methods based on the empirical psychology tradition. While such studies are methodologically sound, they may not be similar to students thinking while sitting in the math classroom. Greater biological validity can imply less controlled protocols, but more related to areas where mathematical learning and thinking occur. De Smedt et al. [29] suggested that research in neuroscience and education should rather be conceived as a two-way street between these two fields of research. Turner (2011) provided a critical examination of the possible connections and argued that there is currently a very unbalanced one-way exchange and dominance of cognitive neuroscience. This unbalanced exchange arises because the limitations of neuroimaging research are not always clearly articulated and because the findings that it generates are often taken for granted and not open to critical evaluation, particularly not by educational researchers. De Smedt et al [29] mentioned that we appreciate Turner's (2011) efforts to undertake this critical examination, especially because, a critical interrogation of cognitive neuroscience from within the field of education is currently missing. Neuroimaging research limitations are highly similar to any quantitative research method. It is inappropriate to assume that the results of one approach are more informative or valued than the results of any other studies. If the field of neuroscience and education wants to move forward, a critical acknowledgement of the strengths and weaknesses of each subfield and mutual respect for both research traditions, that (might) have distinct philosophical backgrounds, are crucial. The connection between both disciplines should not be limited to a one-way street view in which findings from cognitive neuroscience are applied to educational theory. This would be the imbalance that Turner (2011) is pointing to, where one of both partners is dominant. Educational researchers and cognitive neuroscientists should understand each other more fully than is currently the case, a challenge for the future in which the role of education of researchers both in education and in cognitive neuroscience will be fundamental. Without doubt, neuroimaging research has its limitations, highlighted by the authors. However, although they are very important, they are specific to FMRI and most of which are highly similar to any quantitative research method. These limitations should not reflect a negative view of advances in cognitive neuroscience, it is important to note that the appropriateness of these methods depends on the research question at hand. When a researcher is interested in very specific low-level processing, cognitive neuroscience data allow one to understand learning at the biological level. These methods can help to measure processes that are difficult to access by means of behavioral techniques and more indirectly, neuroimaging data can be used as an input for research on learning and instruction [20].

Application of numerical processes to mathematical achievement

In a meta-analysis, De Smedt et al. [6] showed that there is a growing literature describing that the numerical skill in children's processing symbols strongly predicts future mathematical development. It is not clear, however, whether numeric symbols are simply placed on large (non-symbolic) symbols or whether counting sequences play a crucial role in understanding these symbols. This question was addressed in an EEG study by Merkley et al. (2016), which used an artificial symbol-learning paradigm in which the adults acquired a new and novel number symbol using non-symbolic magnitude information or sequencing. They found that both EEG patterns in processing new symbols were similar after practice, suggesting that individuals could create representation of number symbols based on the amount or order of information. These findings confirmed the positive role of ordinality as an important facet in the development of mathematics skills. Using FMRI analysis, Vogel et al. (2016) and Schellinger et al. (2016) examined the role of underlying factors in number processing, suggesting that numerical information processing may change depending on the context, and this may vary based on the level of one's ability to communicate. In another study, Poolak et al. (2016) analyzed how the processed electronic symbols can be used in mathematics to represent values; their findings opened up a new way of investigating the subjective representations used in higher-order mathematics such as algebra. Neurological cognitive research by Dehaene and Amalric (2016) revealed a link between the numerical processing and relatively advanced mathematics thinking. The researchers examined the neural origins and implications of mathematics expertise. They used FMRI for 15 professional mathematicians and 15 humanities specialists with similar educational backgrounds. The participants were asked to judge the accuracy of mathematical and non-mathematical statements. A direct comparison of the groups revealed that parietal and frontal activation during reflection on mathematical statements was only present in the group of expert mathematicians. The experiment demonstrated that the brain regions employed by expert mathematicians during their reflection on mathematical statements are located outside areas typically associated with language. The findings contradicted previous findings of studies on numerical cognition, which had demonstrated connections between activation evoked by numerical processing and by language. The research by Amalric and Dehaene (2016) shows that language may play a role in the initial acquisition of mathematical competencies and that brain activation during elementary numerical processing and higher level mathematics are connected; they thus demonstrated that advanced mathematical processing is connected to symbolic number processing. The connection between advanced mathematical processing and number sense can develop an awareness of the importance of nurturing mathematical minds from the early stages of development. Additionally, these findings can lead to a hypothesis stating that early competencies associated with number processing and numerical operations can constitute predictors of later mathematical expertise and, probably, of mathematical talent. Gonzalez-Gardido [19] illustrated childhood differences in mathematics achievement by analyzing EEG cohesion and brain function dynamics compared to symbolic values. The results showed that different mathematics skills represent varying degrees of number processing system, possibly involving several complex neural networks with distinct topographic distribution. It is mainly affected by post-encryption processing steps probably encouraging the selection of more appropriate cognitive strategies for available processing resources. However, further studies need to be conducted to determine whether the functional connections related to the numerical comparison task are directly correlated to other factors that contribute to the general factors involved in mathematics learning such as working memory, reasoning, and processing speed. Geary [21] suggested that the early processing and manipulating numerical set size and Arabic numerals, the accurate employment of advanced counting methods to solve the addition problems and accuracy in making placements on a mathematical number line were uniquely predictive of mathematics achievement. He concluded that the early quantitative competencies uniquely contribute to mathematical learning.

Students' academic achievement in mathematics using brain-compatible learning

Azalia Herma [14] discusses the reasons of students who are not interested in mathematics: 1) the school is taught math fun as properly. 2) Students are more fixated with the formula, instead of the embodiment process or phenomenon generated by mathematics. 3) When work on the problems related to mathematics, they are no longer as critical thinking, but think of the crisis; so it is clear what we cannot understand, cannot be enjoyed. 4) The present school use of force so that students want to learn mathematics; this has resulted in the emergence of fear, anxiety, and do not like in the students when faced with problems. She believes that there is a relationship between the brain understanding and students' perception so that understanding how the brain processes the data, teachers and education practitioners can understand how interesting teaching methods and can increase students' interest in students. For example, using games can stimulate the right brain works and maximize work dorsolateral prefrontal cortex, producing a deep impression and opened the paradigm of mathematics. Roza Leikin [7] suggests that neuroimaging studies demonstrate the neural correlates of mathematical difficulties and disabilities. At the other end of the continuum, research has also demonstrated connections between intelligence and brain activity related to different cognitive tasks. Neuroimaging research shows that intelligence is associated with the reciprocity of several brain regions within a widespread brain network. Another branch of neurocognitive research focuses on the relationship between intelligence and the extent of induced brain activity during cognitive task performance. These studies have led to the formulation of the neural efficiency hypothesis of intelligence, which states that brighter individuals display lower (more efficient) brain activation while performing cognitive tasks. At the same time, task difficulty has an effect: The neural efficiency phenomenon is revealed in easy to moderately difficult tasks, whereas when it comes to performing difficult and challenging tasks, more intelligent individuals exhibit higher brain activity. Azalia Herma [17] maintains that understanding of cognitive neurosciences will help teachers to change the paradigm of people about mathematics and develop the interest of students to deepen the field. They employ the learning methods delivered in an easy and fun way, namely through demonstration, game, and introduction of interdisciplinary study. The advantages of understanding above are the paradigm of society, especially students, got easily changed. In addition, the development of a person's cognitive abilities can be maximized, not only in mathematical processing, but also in many fields. Seifi et al. [21] suggest that the brain-compatible learning focuses on how the brain learns naturally and seeks to change the framework based on the real structure and function of the human brain to provide effective training. Today, success in life depends largely on students' ability to plan their time, organize and prioritize information and problem solving, all of which are among the components of the brain's executive functions. Historically, executive functions is a general term synonymous with frontal lobe function, where it plays an important role in shaping the fundamental development of cognition and social development; it is the set of cognitive abilities such as planning that allows one to solve a problem or accomplish a goal. It is important to note that the ability to plan for problem solving is essential to mathematics skills. On the other hand, the enriched process of brain cellular connections as well as its flexible ability for the problem-solving process through watching videos and related images by the students provide the necessary impetus for problem-solving skills and in addition to spontaneous enjoyment and effort, they develop problem-solving skills to deepen learning as well as enhance problem-solving skills by providing challenging opportunities in learning situations based on the relaxed alertness.

In an experiment by Juanita M. Costillas [34] investigating the teaching and learning process based on the three basic fundamentals of brain-based training, namely, relaxed alertness, orchestrated immersion, and active processing, it was concluded that the brain-based training facilitates critical thinking both in the inference and maintenance stages. Brain-based training also accelerates stable skills toward critical thinking. Therefore, brain-based training supports the assumption of constructivist theory that a person conceptualizes his or her own understanding based on the experiences. Therefore, brain-based training is able to create assisting experiences towards critical thinking. Similarly, brain-based training verifies the theory on multiple intelligences (MI) having shown that each part of MI works using the brain and that in brain-based training, students had the opportunities to construct their ideas. While noting the importance of visual mathematics, Boaler and Chen [42] explain that new evidence for brain function can change the way mathematics is presented in the classroom while thinking mathematics. In addition to changes in teachers' teaching methods, students should be asked to present their mathematics ideas in a variety of ways, such as pictures, models, diagrams, and even cartoons. In an investigation into the impact of brain-based learning on attitude toward math lesson and academic achievement, Mehdi Hasani et al. [27] concluded that students are able to develop their math lesson by presenting challenging teaching methods and maintain their positive attitudes and tendencies towards them. Academic achievement in mathematics and positive attitudes towards are created in a brain-rich environment for learning (rich learning environment, emotional regulation, and information processing). It also increases one's mathematical learning ability and performance to understand and perceive concepts through the emotional and attitudinal aspects of mathematics education. Improvement of attitude towards the mathematics increases academic achievement and enhances knowledge structure and information processing process in students. A positive attitude towards math stimulates brain activities such as thinking and learning, and ultimately, sustainable learning takes place through creating positive attitudes towards mathematics classroom. As a mathematics teacher, Connie White is interested in helping his students understand how they learn and empower them to control over their learning. She found research about the brain can catalyze our understanding of how students learn and how teachers, in response, should mobilize. According to the dimension of "changes in math teaching through understanding brain function while learning math", by creating a positive environment, endorphins are released in the student's brain and stimulate the front lips leading to a successful and enjoyable learning experience for the student. Students' awareness of brain flexibility process may help him/her to develop an understanding of neural circuits functioning. As neural pathways become stronger or reinforced, information is stored better and can also be easily retrieved. If students consistently memorize the facts, their memory paths will become stronger. Therefore, as students learn to think, analyze and solve problems, they become more proficient in critical thinking. Incorporating different memory paths will improve learning and leads to easier retrieval. The brain's requirement for multiple experiences such as multi-sensory inputs, scaffolding to previous learning and the like will be met by activating auditory, visual, and kinetic strategies. Students who have the opportunity to simply reflect on what they have done, have a deeper learning experience and their memory will be strengthened. Students are able to make a big difference in their math classroom by deliberate planning at their frontal lope. Awolola [35] compared the brain-based learning strategy with teaching technique that centered on explain – practice – memorize. The results showed that the brain-based training strategy enhanced the students' achievement in mathematics more than the conventional lecture method. In another study, M. Macarena (2017) stated that one of the key factors affecting learning is to provide students with the flexibility to improve their brain's potentiality. The findings showed that implementing a brain-based learning approach can improve student achievement and motivation for mathematics learning.

While suggesting the emerging role of educational neuroscience in education reform, Zadina [28] believes that the Educational Neuroscientist offers broad interventions based on Educational Neuroscience that could reform curriculum, and emerging ways the educational neuroscientist can inform new avenues for professional development of educators. A brain-based teacher teaches with a continuous professional development requires the student interaction to achieve their desired goals in a two-way process. For example, exposure to second language learning, music education, proper sleep patterns, meditation and physical education can help promote working memory and increase student attention leading to advanced math skills.

An extensive model for mathematical processing

Roza Leikin [7] states that the neuroimaging research focuses on the underlying brain structures (the magnitude of brain activation as well as brain topographies) associated with different types of mental activities in different population groups. A variety of neuroimaging techniques allows researchers to obtain high-quality information on both temporal and spatial brain activity associated with different kinds of information-processing, including mathematical processing at different levels in individuals with varying levels of abilities. For example, the event-related brain potentials (ERP) technique offers high temporal resolution over the course of problem solving due to a precise reflection of perceptive and cognitive mechanisms. ERPs are electrophysiological measures that reflect changes in the electrical activity of the brain in relation to external stimuli and/or cognitive processes. These measures provide information about the process in real time, before the appearance of any external response. Another major technique is functional magnetic resonance imaging (FMRI), which offers high spatial resolution and enables the detection of differences in processing that are not evident from behavioral and ERP measures alone, thereby potentially leading to a more comprehensive understanding of the underlying processes and brain structures involved. As mentioned, neuroimaging research focuses on localization of brain activation associated with mathematical processing and its relationship to general cognitive abilities (e.g., memory and attention). One example can be seen in the triple code theory of numerical knowledge, which emphasizes the role of the parietal cortex in number processing and arithmetic calculations and identifies three regions of the parietal cortex that have been linked to the different functions connected to number processing. The horizontal intraparietal sulcus has been found to be involved in calculations; the posterior superior parietal lobule (PSPL) has been linked to the visuospatial and attention aspects of number processing; the angular gyrus (AG) has been found to be involved in the verbal processing of numbers and in fact retrieval. Additionally, the parietal cortex has been found to be associated with word-problem solving, algebraic equations, and geometry proof generation. Another example can be found in studies that show that the posterior superior parietal cortex is involved in visuospatial processing, including the mental representations of objects and mental rotations, while the frontal cortex has been linked to attention-control processes and working memory.

Research on mathematical problem solving associated with different representations of mathematical objects is also a focus of neuroscientists. For example, different brain areas are known to be involved in recalling different representations of the functions (verbal vs. equation representations) and are thus connected to different cognitive processes involved in the corresponding mathematical processing. She also argued that while mathematics education is not well informed by neuroscience research, and findings of mathematics education research are rarely used in neuroscience research, the integration of the two research areas can empower each of them. Cognitive research in mathematics education has a variety of foci of attention and research methods. These studies include, but are not limited to, learning and understanding of mathematics as related to problem solving, proofs, proving and argumentation, and defining and exemplification. Special attention is given to investigation and modeling activities, while substantial attention is devoted to difficulties and misconceptions, as well as to expertise, creativity, and giftedness. Schoen Feld (2000) highlighted two main aims of research in mathematics education: one pure and one applied. The pure purpose is related to the understanding the nature of mathematical thinking, teaching, and learning and the second one is related to understanding of improved mathematics education, which ultimately helps to realize math giftedness promoting mathematical creativity. He also emphasized that the empirical or theoretical work in mathematics education should have descriptive and explanatory powers, scope allowing for reproducibility. According to the dimension of "establishing a relationship between the cognitive neuroscience and mathematics education in the classroom using neural imaging", De Smedt et al. [20] identified three types of neuroscience applications to education: neuro-understanding, neuro-prediction, and neuro-intervention. Neuro-understanding is based on the capacity of neuroscientific research to deepen understanding of mathematical processing at the biological level and thus to inform mathematics education theories regarding typical and atypical development of mathematical competencies. Neuro-prediction opens opportunities to use neuroimaging results to predict learning trajectories. Neuro-intervention includes both (1) the use of brain imaging data to analyze the impact of education on the neural circuitry underlying development of mathematical knowledge and (2) the effect of neurophysiological interventions on mathematical performance or learning. An interesting connection between the two fields of research can be seen in the parallel between Schoenfeld's (2000) call for the explanatory and predictive powers of the theories in mathematics education and the neuro-understanding and neuro-prediction types of neuroscience applications to education. In turn, neuroscience has a strong potential for increasing the explanatory and predictive powers of mathematics education theories as well as examining the power of different educational interventions using neuro-intervention Type 1 mentioned above. Obviously, neuroscience research on mathematical processing and cognitive research in mathematics education are complementary. They have many features in common, and each field can provide information that cannot be attained by research methodologies in other fields. Regarding the role of working memory in mathematics learning, Raghubar, Barnes, and Hetch suggest the likelihood that for any individual it will depend on a wide range of factors that influence how the individual interacts with the mathematics information. These include individual factors such as age and skill level, mathematical content factors, and teaching-learning contextual characteristics such as level of goal fluency (elementary, generalized or automated), instruction language and the formats in which the mathematics information is provided. They noted the need for a sufficiently comprehensive model of mathematical processing, particularly in relation to the skill acquisition that can manage current findings in working memory, as well as provide a basis for guiding new discoveries and informing practice. In conclusion, they paid particular attention to the lack of relevant knowledge. The authors noted the need for a mathematical processing theory involving the discovery and selection of strategies, application of mathematical knowledge, and specific aspects of working memory. Contemporary developments in neuroscience have enabled to develop this theory. Teachers and schools can make a valuable contribution to such a model. Teachers observe how their students respond to their given mathematical instruction on a daily basis. They develop at least a visual awareness of the impact of the working memory process on the students' mathematics understanding in a way that most researchers from other disciplines reject. Gaining an insight into how students respond to regular classroom instruction is potentially useful information.

Providing a brain-based training package to help math teachers for teaching

Numerous researchers including Seifi [21] and Mekarina and Ningsih [12], have examined the impact of brain-based learning strategy on achieving mathematics goals and concluded that applying brain-based strategy enhances students' achievement in mathematics. For example, Ramakrishnan [49] explains that brain-based education is the purposeful engagement of strategies that apply to how our brain works in the context of education. Brain-based learning has been called a combination of brain science and common sense. Brain-based learning activities engage both hemispheres of the brain simultaneously, resulting in stronger, more meaningful learning experiences and permanent brain connections. Caine and Caine [50] developed twelve principles that apply what we know about the function of the brain to teaching and learning. The principles are: 1: The brain is a parallel processor, meaning it can perform several activities at once, like tasting and smelling. 2: Learning engages the whole physiology. 3: The search for meaning is innate. 4: The search for meaning comes through patterning. 5: Emotions are critical to patterning. 6: The brain processes wholes and parts simultaneously. 7: Learning involves both focused attention and peripheral perception. 8: Learning involves both conscious and unconscious processes. 9: We have two types of memory: spatial and rote. 10: We understand best when facts are embedded in natural, spatial memory. 11: Learning is enhanced by challenge and inhibited by threat. 12: Each brain is unique. Educational techniques related to brain-based learning include:

1- Orchestrated immersion:
Creating learning environments that fully immerse students in an educational experience. This implies creating an environment where a student feels like he/she is a part of the process and is living it. Teachers must immerse learners in complex, interactive experiences that are both rich and real. One excellent example is immersing students in a foreign culture to teach them a second language. Educators must take advantage of the brain's ability to parallel process.

2- Relaxed alertness:
Trying to eliminate fear in learners, while maintaining a highly challenging environment. Relaxed alertness is the idea of keeping a student's fear in check while still providing a challenging environment. Students must have a personally meaningful challenge. Such challenges stimulate a student's mind to the desired state of alertness.

3- Active processing:
Allowing the learner to consolidate and internalize information by actively processing it. Active processing is the means by which a student is given the opportunity to continually and actively process information to internalize, consolidate, and relate it. In order for a student to gain insight about a problem, there must be intensive analysis of the different ways to approach it, and about learning in general. They suggest that how the brain works has a significant impact on what kinds of learning activities are most effective. Educators need to help students have appropriate experiences and capitalize on those experiences. Knowing how the brain works best allows educators to create an environment that gives the student a higher probability of success in learning. Using the following brain-based learning principles can improve students' performance in class.

Conclusion: Brain-based teaching and learning can become second nature to you. With careful planning, knowledge of brain research findings, and a little creativity, teachers can offer engaging, brain-based activities that encourage exploration and learning and support learning standards. Teachers and students can build a strong community of learners who see learning as an opportunity to be successful problem solvers while anticipating each new challenge as another exciting adventure.

In a content analysis of research documents, Johnson [11] applied data and information discovered to create a brain-based pure math teacher resource that will help teachers teach the pure mathematics with meaning and understanding. The resource includes a rationale, as well as explanations for the brain based mathematics lesson framework. Teacher presents daily lessons friendly laid out on a thematic unit for the algebraic equations, relations and functions section of the curriculum. The resource provides a brain-based approach to teaching and learning offering teachers with an easy to understand, practical, everyday guide that can easily be implemented into the pure math classroom. This resource is needed because students continue to feel inadequate and inferior in pure math classrooms. Changes needed to resolve this disturbing situation include teachers themselves altering their teaching strategies to help minimize the existing problems in the pure math program, and this project contributes to the knowledge about improving best teaching practices. Research on the multiple intelligence theory reminds us of the different student learning styles and the fact that, more than one type of teaching strategy should be used to deliver the pure math program. Current research on the science of learning has brought to light some very interesting ideas of how a student's brain works and applications of this work to classroom practice. Teachers translate this information into classroom practice in order to help their students learn pure mathematics with meaning and understanding. Previous learned knowledge, attention and learning, emotion and learning, movement and learning, rehearsal and learning, memory and learning, elaboration and learning, and collaboration and learning. Each lesson encompasses activities that call for attention, rehearsal, elaboration, problem solving, project wok in real life situations, and reflection, sharing, writing. All of these ideas connect together to produce a teacher resource that changes the presentation of the pure math 20 unit on quadratic equations and functions to its students in order to teach pure mathematics with meaning and understanding. From what has been discussed above, we can conclude that while the objective of extracting a model for the continuous professional development of mathematics teachers based on the brain-training approach is to support teachers to teach better and consequently students to learn more effectively, brain-compatible learning emphasizes how the brain learns naturally and seek to change the framework based on the actual structure and function of the human brain to create effective education. Brain-compatible learning is a complex and informed approach to current educational problems. Researchers working in brain-based learning try to empower students' brains. Brain-based learning is coordinated with brain action and derived from brain understanding. Therefore, it is mandatory to provide a brain-based training package to support mathematics teachers for teaching math lesson.

Summary And Conclusion

Learning is one of the interesting and somewhat difficult areas in psychology and education sciences with diverse applications in different fields of education. Mathematics education is a branch of the humanities that has gained an important position in the world's scientific circles particularly in developed countries in recent years. Therefore, scientific identification of learners' problems in mathematics and planning and making an effort to solve them by teachers, planners and authors based on brain-based learning principles lead to academic achievement in mathematics including qualitative and quantitative changes in developing competent behavior of math students. Major developments have been made in brain understanding, learning and neuroscience research in recent decades. In 2010, a special issue in cognitive neuroscience and mathematical learning was published at ZDM. ZDM is a series of empirical research articles that address key questions in cognitive neuroscience and mathematics learning using multidisciplinary and innovative approaches. A survey of the current articles on the issue reveals that there has been growing interest in this field recently and more complicated questions have been addressed using a wide variety of methods including eye tracking, EEG, and functional neuroimaging. In an effort to connect cognitive neuroscience research with mathematics education issues, it is clear that there is great potential for specific research projects and extensive knowledge exchange between the two fields. These studies have consistently cited research limitations including sample selection and biological validation. It should be noted that these limitations are not specific to cognitive neuroscience. They are clearly evident in much behavioral and psychological research as well. It is important to pay special attention to these limitations; however, the main subject should not be called into question. Compared to educational research, cognitive neuroscience is still very young as a field, with the assumption that it quickly will resolve key issues and questions in (mathematics) education, which have been there for quite a long time. Just as any scientific approach, it is a gradual process that is marked by conjectures and refutations as well as advances in measurement and analysis. The ultimate aim of the learning sciences is to provide a multilevel analysis to understand how learning takes place and how it can be fostered, with each level of analysis (e.g., behavior, cognition, brain) being compelling in its own right. One of the concerns in this field is how to apply the results of mathematics neuroscience research in the classroom. Therefore, this study moves towards providing a model for the continuous professional development of mathematics teachers. Given the increasing number of research in the area and the proven effectiveness of brain-training based approach to mathematics in the classroom on the student learning, it is time to create an interaction between the mathematics teachers and interdisciplinary researchers, because interdisciplinary teacher education can act as a critical mechanism for promoting and developing student learning. A close examination of numerous studies shows that the possible pathways in which mathematics education and educational neuroscience can be invested have been identified [7]. Mathematics education can help to design research process, while the neuroscience can support theories in mathematics education and further the interpretation of research results. This will not be realized unless the mathematics teachers continue to professionally develop their neuroscience of mathematics education and cooperate with neuroscientists in order to set the conditions for integrating the two areas of study (neuroscience and mathematics education).

Administrative recommendations of the study

- The findings of this study can provide a basis for continuous professional development training of mathematics teachers so that they can move toward improving the students' mathematics learning. Today, there are several foundations for teacher education, which are primarily focused on subject knowledge.
- The dimensions of this model, especially "establishing the relationship between the cognitive neuroscience and mathematics education in the classroom with the help of neuroimaging" and the dimension of “role of working memory and attention control in mathematics achievement ", can serve as a basis for providing a brain-based training package to help math teachers in teaching. These methods can help them make some necessary changes in methods of teaching mathematics through understanding the brain function while learning math resulting in students' academic achievement in mathematics using the brain-based learning approach.
- Educational policymakers are the key contributors of the study. Given the dimensions of the model presented in this study, they can adopt a new approach to decision making and set necessary conditions for making changes in teaching methodology and student learning.

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