# The Effect of Manipulation on Reducing the Errors of Fourth-Grade Elementary Students while Solving Fraction Word Problems

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## Abstract

The main aim of this study was to investigate the effect of manipulation on reducing the errors of fourth-grade students while solving fraction word problems based on Newman error analysis (NEA) model. Participants in this study included 48 female students from two elementary schools; regarding which, two fourth grade classes were selected by cluster sampling. In this study, it was attempted to evaluate the effect of teaching fraction and word problems through manipulation on reducing the errors of students and to classify their errors (reading, comprehension, transformation, processing skills, and encoding) according to the NEA structured model. Therefore, the study design was quasi-experimental by pretest and posttest with a control group. Data collection tools were standardized and researcher-made tests together with Newman's qualitative interview. The content validity of questions was obtained by the CVR method and the reliability was validated by the Kuder-Richardson formula. Data were analyzed using descriptive and inferential statistics. Results showed that teaching mathematical word problem solving through manipulation led to significant differences in reducing the errors of students.

**Keywords:** Manipulation, Students' errors, Word problems, Newman Error Analysis Model

## INTRODUCTION

Educational systems mainly aim to provide students with the necessary skills to play an effective role in the growth and development of society. Given the present-day circumstances, mathematics plays a significant role in presenting and transferring these knowledge and skills [1].

Among different aspects of mathematical practice, problem solving is of special importance and the vast majority of mathematicians and teachers believe that the most important factor in learning mathematics is the problem solving ability [2]. Among mathematical problems, word problems have long been of a special interest [3]. Most researchers consider that word problems should be included in the school mathematics curriculum for the development of students' potential abilities. Word problem solving accounts for an important component of mathematics problem solving that encompasses real-world problems and their applications [4].

In order for students to learn mathematics with comprehension, an important point is that they should actively participate in their own learning (NCTM, National Council of Teachers of Mathematics, 2000) [5]. According to Yusof and Lusin (2013), students should actively involve in learning to create better comprehension of mathematical concepts by manipulating objects in their surroundings [6]. In ideal teaching and learning of fraction, therefore, students are provided with the opportunity to explore the concept of fraction through hands-on experiences. Post (1980) extended this idea by confirming that manipulatives are tools or things that can engage multiple senses, meaning that students are able to sense them [7]. This simplifies situations from the real world, and simultaneously symbolizes abstract concepts [8].

Symbolization in the mathematical world allows students to make calculations and introduce concepts to attain predictions that can be confirmed in the real world [8]. When

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students interact with objects, they take the first step toward understanding mathematical processes and methods. Effective use of manipulation can help students to connect ideas and amend their knowledge so that they gain a deep understanding of mathematical concepts [9].

In the mid-1970s, an Australian linguist, called Newman, introduced a systematic method of analyzing errors made by students when answering written mathematics questions. To analyze the errors of students in solving word problems in this structured model, the learner has to go through the following order to answer a word problem: 1) Reading, 2) Comprehension, 3) Transformation, 4) Process Skills, and 5) Encoding (rewriting answer in written form) [10].

Fractions and fraction of word problems are one of the most important outlines of mathematics and one of the most useful and abstract problems in the real world of mathematics. Considering the importance of fraction word problems in the fourth grade, as well as the identification of challenging areas in solving this type of problems, little research has so far focused on this field in Iran. To provide appropriate solutions (the use of manipulation) to moderate errors in this category, this study attempted to investigate the ability of fourth-grade elementary students in solve fraction word problems and their possible errors in the area of using manipulation.

## Theoretical Background

### Manipulation and manipulatives

Today, all science education experts strongly believe that teaching is effective if "students learn through first-hand experiences, direct experiments, and engage with teaching aids, research, and problem solving" [11]. According to Bezuk and Cramer (1989), the use of manipulation is an important strategy to help young students develop mental images of the part-whole meaning of fraction [8].

Kilpatrick et al. (2011) also suggested that teaching can help learners to improve their mathematical knowledge provided that they spend time in practicing and acquiring necessary skills [12]. Since the teacher is the motivator of any educational activity, he/she should be familiar with more teaching methods to have more freedom toward actions to achieve educational goals in different educational situations [13].

Accordingly, manipulatives are objects or things that students are able to feel, touch, handle, move [14], and often collect [8]. According to [14], manipulatives are effective and motivating tools for helping and improving the development of mathematical concepts. Children can physically manipulate these objects, and, when used properly, they can provide children with the opportunity to measure relative sizes of objects that can express mathematical ideas such as fractions or place value. They also allow children to identify patterns and combine number representations in several ways.

Manipulatives appear in a variety of forms and are often defined as "physical objects used as teaching tools to engage students in practical mathematical learning" [15]. Manipulatives, therefore, can be purchased from a store, brought from home, or made by teachers and students. Smith (2009) stated that a well-structured manipulative can fill the gap as a bridge between formal and non-formal mathematics [16].

### The relevance and importance of manipulation in mathematics education

In order to prepare and manipulate students, they need to actively experience mathematics with different representations, and teachers can use different tools as part of their mathematics education. Real objects, such as manipulation in mathematics, are tools that can help students better understand mathematical concepts [17], and provide students with the opportunity to visualize and maneuver objectively (realistic) for abstract mathematical concepts. Students, therefore, are provided with a mechanism to communicate with the symbolic language of mathematics to have better understanding of the reality.

Manipulatives are important tools for learning mathematical concepts because they act as mediators by helping students to fill the gap between their understanding of objective models and abstract concepts [18-21]. Post (1980) [7] and Meira (2002) extended this idea by confirming that manipulatives are sense-making tools that simplify circumstances from the real world, and simultaneously symbolize abstract concepts. Symbolization in the mathematical world enables learners to make calculations and exemplify concepts to achieve predictions that can be affirmed in the real world [8].

Accordingly, Smith (2009) claimed that there are as many inaccurate methods of teaching with manipulatives as there are teachings with no manipulations. The mathematical manipulatives should be suitable for students and be selected to fulfill the specific aims and objectives of the mathematical program. Stein and Bovalino (2001) also pointed out that "if manipulatives are not used accurately in teaching, then they become smaller than objects behind the showcase that are only good for looking and playing, but are unnecessary for learning". The use of manipulatives can be appropriate in an environment where the teacher uses them as a technique to encourage thinking about a subject.

### Word problems and errors in solving word problems

Mathematics education researchers have provided plentiful but similar definitions of word problems. For example, Adams (2003) has defined a word problem as a mathematical exercise expressed in the format of a story or a text derived from facts. He pointed out that a student needs to acquire skills to interpret the information presented in the text into mathematical symbols [3].

Palm (2009) considered a word problem as a mathematical exercise veiled in a real-world situation that students need to unveil these exercises and solve the problem [3]. According to Lave (1992), word problems are a special type of mathematical problems that describe a situation of the real world. Students need to use mathematical operators and data in the problem text to solve word problems. These problems are an important part of a school mathematics curriculum provided with the aim of applying formal mathematics skills and knowledge to the real-life situations of students [22].

According to Clements and Ellerton (1993), the importance of word problems lies in the language centricity for teaching and learning of mathematics. Working on word problems also helps develop one's awareness of the real world [23]. According to Kyle Patrick, Swaffard, and Findell (2011), solving word problems enables learners to understand how mathematics can play a role in their real world [3]. However, teachers seem not to be interested in mathematical word problems as they think that such problems are not suitable to evaluate students and misleads them [23]. Montague (2006) has defined word problem solving in a two-step process of problem "representation" and "problem execution", both of which are necessary for successful problem solving [3].

As stated by Meyer (1992), the first step in solving word problems, i.e. understanding the text of problem, is done through translation and integration. In the translation process, the problem has to be extracted from the word form and take the form of an internal representation. However, the word-by-word and phrase-by-phrase translation of the problem text creates a fragmented representation of the semantic structure of the text that is understood in the integration process as an integrated structure. He defined the second step of solving word problems as the solution execution, which is programmed and executed using appropriate mathematical algorithms, leading to an answer, whose suitability is examined as a solution. These processes are highly consistent with the problem-solving model of Polya [22].

According to Koedinger (2004), students should be allowed to solve problems by various ways, such as drawing, using diagrams, pictures, and vocabulary. To solve word problems at different educational courses, students also require linguistic knowledge to represent and understand the situation described in the problem in addition to mathematical knowledge. The way of expressing mathematical word problems is one of the factors that influences the process of understanding and solving word problems [24]

## Students' difficulties in solving fraction word problems

Students usually make mistakes in solving fraction problems [25]. Hasser and Abouze (2003) studied individual differences of students in the use of calculation concepts and procedures for fractions in relation to procedural knowledge as students' awareness of the process steps required solving a problem, as well as conceptual knowledge, which includes the relevance of concepts to mathematical symbols [26]. Their results revealed that students' conceptual and procedural knowledge led to the selection of different strategies for solving word problems, including fraction [27].

Clemens (2004) believed that students have a hard struggle with mathematical word problems. At the elementary level, most of their mistakes in mathematics tests and exams result from reading and comprehension errors. This means that most students are able to perform one or more of four fundamental operations but do not know which operation they can use [23]. Research has also shown that students become confused when computational algorithms and procedures are presented as word problems [28], because students' comprehension of these procedures and algorithms are procedural rather than conceptual. As stated by Liu et al. (2012), students do not understand the concept of algorithms and fraction correctly, though they can easily apply algorithms.

Axou (1997) stated that students perform best in operations with fractions and computational questions and have the poorest performance in solving word problems. In a study, Hasser and Aubois (2003) examined the fifth-grade students' comprehension of fraction word problems and found that students chose different approaches to problem solving [26]. Besides, students' inaccurate solutions often result from a lack of problem understanding and of misunderstanding the part-whole concept and operations with fractions [29].

## Errors in solving word problems

Loventa (2008) stated that "errors are natural and justified attempts occurring in understanding mathematics. This is particularly evident when students actively try to make their experiences meaningful by linking school knowledge and everyday events, such as learning fraction and linking it to the concept of division at home" [30].

The person looking for the correct answer to word problems will eventually act according to the following hierarchy:

1. Read the problem.
2. Understand what is read.
3. Change the question words in his/her mind to the choice of strategy.
4. Apply the process skills required by this selected strategy.
5. Encode the answer in an acceptable written form.

Newman (1983) suggested that the following questions and queries can be asked in the interview to categorize students' errors in their mathematical tasks-writings (ibid, 2014).

1. Please read the problem to me (reading)
2. Tell me what the problem is asking you to do (comprehension)
3. Tell me how you are going to find the answer to this problem (transformation)
4. Show me what to do to get the answer to this problem. Explain it while doing this (processing skills)

5. Write the answer to this problem now (encoding)

In addition, Prakitipong and Nakamura (2006) considered success in the NEA initial two steps (Reading and Comprehension) as signifying the fact that learners have correctly interpreted the question in the mathematical context [31]. The completion of the final three steps (Transformation, Process Skill, and Encoding) implies that learners have successfully executed the mathematical processes required to solve the task. Findings of Ahmad et al. (2010) suggested that the major error in solving word problems is translating the word representations into mathematical representation. They also pointed out that learners' inability to visualize and extract mental images, results in their inability in mathematical reasoning.

Sepeng (2011) and Vershaffel, Greer and Van Dooren (2009) reported that students tended to eliminate and escape from real-world knowledge when solving word problems [32, 33]. Students can find a relationship between school mathematics and their daily lives. Sheen (2009) argued that the use of pedagogies and educational techniques should be avoided as they eliminate the opportunity to analyze the word problem and the contextual comprehension embedded therein [34]. According to Acosta Tello (2010), the source of students' errors can also be the inappropriateness and mismatch of teaching methods and the way of evaluating students [35].

## Students' comprehension of solving fraction problems

The transition from working with integers to fractions is difficult for students [27]. They attend the classroom with a wide range of informal fraction knowledge. The knowledge that students bring to the classroom is mostly based on the part-whole concept. Researchers have stated that this informal understanding is a good basis for developing students' conceptual comprehension of fractions [36]. One of the approaches offered by the NCTM (2000) to improve teaching fraction is to use physical objects and models existing in the real world [5]. These standards emphasize the link between classroom mathematics and out-of-class mathematics; to deepen this connection and also make it visible to students, students should be provided with problems from the real world surrounding them [37].

Mack (1993) also highlighted the limited informal knowledge of students. In other words, students' informal strategies relate to fraction problems, such as whole number problems partitioning, which affect students' informal comprehension of fractions and their ability to re-understand the unit, and their informal knowledge is initially unrelated to their knowledge of formal symbols and fraction-related procedures [36], For example, a student was presented with a problem; a person was given one-eighth of a pizza and got one eighth more, what is the total amount? The student answered that it would be two-sixteenths since there were two pizzas, one whole pizza with eight pieces and another whole pizza with eight pieces [27]. This suggests that students' informal knowledge of fractions is mostly based on whole-number strategies, often resulting from incorrect answers. Also, a look at students' correct answers reveals that they choose different ways and solutions to the problems. Incorrect answers of students are also valuable and informative as they reflect the thoughts of such students and can also reflect their misunderstandings [27].

## Methods

The research design was quasi-experimental with one test group and one control group using pretest and posttest procedure. Students from the two selected classes were randomly assigned to control and test groups. A test with interview was first used to analyze the known errors of students in fraction word problems in both control and test groups. After examining students' errors, the test group received teaching fractions by manipulation in 11 sessions of 45 minutes. After the end of teaching sessions, the number of errors made by fourth-grade elementary students in solving faction word problems was re-measured with posttests and interviews based on the NEA model. The statistical population of this study included all fourth-grade elementary female students (n = 1160) in District 6 of Isfahan during the academic year 2016-2017. The students were sampled by multistage cluster sampling method, and a sample size of 48 students was assigned to control and test groups.

The research consisted of 20 sessions, 11 of which, with a mean duration of 45 minutes belonged to teaching, and the rest was considered for pretests, posttests, and interviews. The research process was initiated with a written pretest (comprising five questions of fraction word problems) consisting of Thames, valid paper, and researcher-made questions to measure students' errors in solving fraction word problems in both groups. The students' errors were analyzed after they were interviewed according to the NEA model. The error analysis revealed that students made most of errors in the comprehension, transformation, and process skills stages and the least errors were in the reading and encoding stages. Therefore, reading and coding errors were excluded from the study, and the impacts of manipulation were examined on three stages of comprehension, transformation, and process skills.

After determining the research process, teaching was initiated by manipulating the concepts of fractions including fraction concept, mixed numbers, equivalent fractions, fraction comparison, fraction simplification, add and subtract fractions, and multiplication of fractions.

In the control group, the concepts of fractions and fraction word problems were taught with a traditional approach. Teaching the concepts began using illustrations, textbook activities, and lectures, and several word problems related to the subject were taught to the students at the end of each session to review their mistakes and misunderstandings in solving the problems.

The validity of pretest and posttest was evaluated through the content validity ratio (CVR) method. Pretest and posttest questions were evaluated separately by 15 experts, consisting of four mathematics faculty members and 11 teachers experienced in fourth grade. Of 15 questions at each stage (pretest and posttest separately), five questions were selected with an acceptable minimum CVR and used for pretest and posttest procedures.

The pretest reliability was validated by the Kuder-Richardson formula and Cronbach's alpha, each with the values of 0.78 and 0.74, respectively. Posttest reliability was also validated by the Kuder-Richardson formula and Cronbach's alpha, each with 0.87 and 0.85, respectively.

The concepts of fractions and solving word problems related to each concept were taught by the instructor (researcher). Then, both groups were subjected to a written posttest (having five questions of word problems) using the same questions as the pretest, with predetermined validity and reliability. After doing the Newman's interview, pretest and posttest scores of the two groups were analyzed using statistical software to test the research hypotheses.

## Findings

To use parametric tests, Table (1) represents the results of Levene's test for the equality of error variances.

**Table 1.** Levene's test for the equality of error variances

| Variables | F | 1st df | 2nd df | Sig. |
|---|---|---|---|---|
| **Comprehension error** | 3·109 | 1 | 46 | 0·084 |
| **Transformation error** | 2·825 | 1 | 46 | 0·100 |
| **Process skill error** | 13·541 | 1 | 46 | 0·001 |
| **Total score** | 3·585 | 1 | 46 | 0·065 |

Table (1) shows Levene's test the equality of error variances, indicating the equality of variances for the errors of comprehension and transformation but not for process skills.

**Table 2.** Kolmogorov-Smirnov test for total error score

| Variables | Group | Statistic | df | Sig. |
|---|---|---|---|---|
| **Total error score** | Test<br>Control | 0·151<br>0·169 | 24<br>24 | 0·162<br>0·074 |

The results of the Kolmogorov-Smirnov test indicated normal total error scores (Table 2).

### Main question

Does manipulation has an impact on the reduction of errors in fourth-grade elementary students to solve fraction word problems based on the NEA model?

**Table 3.** Analysis of covariance for total score of errors in fourth-grade elementary students

| Source | Sum of squares | df | Mean square | F | Sig. | Eta value | Power |
|---|---|---|---|---|---|---|---|
| **Pretest** | 29·534 | 1 | 29·534 | 33·288 | 0·001 | 0·425 | 1·000 |
| **Group** | 27·822 | 1 | 27·822 | 31·259 | 0·001 | 0·411 | 1·000 |

As shown in Table (3), the groups were significantly different in the total score of errors in the posttest ($P < 0.05$). The significant difference between the total error score of fourth-grade elementary students in solving fraction word problems based on the NEA model in the test and control groups at the posttest stage indicated that teaching by manipulation could reduce the total score of these students' errors.

**Table 4.** Results of multivariate analysis of covariance (MANCOVA) for differences between the test and control groups concerning errors of students

| Source | Value | F | Sig. | Eta value | Power |
|---|---|---|---|---|---|
| **Group** | 0·323 | 6·506 | 0·001 | 0·323 | 0·956 |

The results of MANCOVA (Table 4) displayed the difference between the test and control groups in terms of error types for fourth-grade students in solving fraction word problems based on the NEA model. Accordingly, a significant difference was observed between the test and control groups in the variable of students' errors in solving fraction word problems based on the NEA model ($p < 0.01$). An Eta value of 0.323 explained 32.3% of the difference between the two groups in the error types of fourth-grade elementary students in solving fraction word problems based on the NEA model. Also, a test power of 0.956 indicated the adequacy of sample size.

### First sub-question

Can manipulation reduce the comprehension errors of fourth-grade elementary students in solving fraction word problems based on the NEA model?

**Table 5.** Results of MANCOVA for the number of comprehension errors in fourth-grade elementary students

| Source | Sum of squares | df | Mean square | F | Sig. | Eta value | Power |
|---|---|---|---|---|---|---|---|
| **Pretest** | 3·507 | 1 | 3·507 | 12·459 | 0·001 | 0·225 | 0·932 |
| **Group** | 0·611 | 1 | 0·611 | 2·169 | 0·046 | 0·048 | 0·302 |

According to Table (5), the groups were significantly different in the number of comprehension errors in the posttest ($P < 0.05$). The significant difference between the comprehension errors among fourth-grade elementary students in solving fraction word problems based on the NEA model in the test and control groups at the posttest stage indicated that teaching by manipulation could reduce the number of comprehension in these students in the posttest phase.

### Second sub-question

Can manipulation reduce the transformation (translation) errors of fourth-grade elementary students in solving fraction word problems based on the NEA model?

**Table 6.** MANCOVA results for the number of transformation (translation) errors in fourth-grade elementary students

| Source | Sum of squares | df | Mean square | F | Sig. | Eta value | Power |
|---|---|---|---|---|---|---|---|
| **Pretest** | 0·370 | 1 | 0·370 | 1·275 | 0·265 | 0·029 | 0·113 |
| **Group** | 2·944 | 1 | 2·944 | 10·142 | 0·003 | 0·191 | 0·876 |

Based on Table (5), the groups of fourth-grade elementary students were significantly different in the transformation errors of solving fraction word problems based on the NEA model in the posttest stage indicating that teaching by manipulation could reduce the transformation errors in these students in the posttest phase.

### Third sub-question

Can teaching by manipulation reduce process skill errors of fourth-grade elementary students in solving fraction word problems based on the NEA model?

**Table 7.** MANCOVA results for process skill errors in fourth-grade elementary students

| Source | Sum of squares | df | Mean square | F | Sig. | Eta value | Power |
|---|---|---|---|---|---|---|---|
| **Pretest** | 3·286 | 1 | 3·286 | 11·784 | 0·001 | 0·225 | 0·919 |
| **Group** | 1·790 | 1 | 1·790 | 6·418 | 0·008 | 0·215 | 0·697 |

Table (7) represented the significant differences between the groups of fourth-grade elementary students in process skill errors of solving fraction word problems based on the NEA model in the posttest stage suggesting that teaching by manipulation could reduce process skill errors in these students in the posttest phase.

## Conclusion

This study was conducted due to poor performance of students in fractions and fraction word problems to evaluate students' difficulties and errors in solving fraction word problems according to the NEA model and to provide a solution to these errors. The data required to test the research hypotheses and to answer the research questions were collected using resources such as written tests (pretest and posttest) and Newman's interview. The results of the first sub-question have been first discussed below, followed by those of the main and sub-questions.

### Main question

Table (3) showed that the answer to the main question was confirmed by F test for two independent groups (test and control groups) at a significance level of 0.05. In other words, there were significant differences between the mean posttest scores of the test and control groups. It can, therefore, be concluded that teaching by manipulation in solving fraction word problems was an effective method in reducing students' errors based on the NEA. This result supported that of Rastizadeh (2014), who reported reduced students' errors in word problem solving using examples and methods based on the NEA model. The above finding was also in line with that of Leslie White (2010), who used teaching based on an arithmetic calculus program and teachers applied this method as a corrective educational strategy. They used NEA as a diagnostic tool and their results showed reduced students' errors based on the NEA model.

### First sub-question

This question was answered according to the MANCOVA results of comprehension errors (Table 5), showing a significant difference in the number of fourth-grade students' comprehension errors in solving fraction word problems in the posttest of the test and control groups. This means that teaching by manipulation could reduce the number of errors in these students based on the NEA model in the posttest phase, which supported that of Meir, Kay, and Gramp (1997), who reported improved students' mathematical understanding by teaching problem-solving using different approaches [3].

### Second sub-question

This question was answered according to the MANCOVA results of transformation errors (Table 6), presenting a significant difference in the number of fourth-grade students' transformation errors in solving fraction word problems in the test and control groups at the posttest stage. This means that teaching by manipulation could reduce the number of transformation errors in these students based on the NEA model in the posttest phase. This result confirmed that of Clements (2004) on the lack of recognition in elementary students for the correct choice of mathematical operations [38], and also that of Ghorbani Seasakht (2009) in improving problem solving instruction.

### Third sub-question

This question was answered according to the MANCOVA results of process skill errors (Table 6), indicating a significant difference in the number of fourth-grade students' process skill errors in solving fraction word problems in the test and control groups at the posttest stage. This means that teaching by manipulation could reduce the number of process skill errors in these students based on the NEA model in the posttest phase. This finding confirmed that of Khodayari (2013), who reported that teaching problem solving through the rod model could help students overcome their wrong beliefs about inability to solve problems [29].

## Discussion and Recommendations

The findings of the present study demonstrated that the manipulation-based educational approach in teaching fractions could develop students' conceptual and procedural understanding of fraction concepts, balance between their conceptual and procedural perceptions, and make learning these concepts more stable. It also enabled students to combine their prior and current knowledge, solve the problem, and ultimately achieve the highest level of learning (problem solving).

In addition, manipulation-based education represented the problematic data to students tangibly and helped them identify problem-solving processes. The use of this model also linked mathematical concepts together and accelerated teaching problem solving.

It can, therefore, be concluded that manipulation promoted conceptual understanding, provided more stable learning and, consequently, improved students' achievement of the highest learning level, that is, problem solving according to Gagne. It was also effective in reducing students’ errors while solving fraction word problems, enabling them to apply their learning to succeed in problem solving. The findings of this study confirmed the similar results of, for example, Axou (1997), Doosti (2013), Trespalacius (2008), and Rastizadeh (2016).

Accordingly, it is recommended to conduct a similar study on male and female elementary students to compare the results between the two genders. This research was based on discussions related to fraction word problems in fourth-grade elementary education, hence upper grades should also be explored in this respect. According to our observations, the manipulation could eliminate about 0.41% of students' errors in solving fraction word problems. Other researchers are recommended to use other modulatory teaching methods to eliminate other students' errors in fraction word problems. It is also suggested to carry out a research similar to the present study on the other concepts of mathematics such as geometry.

## References


1. Shariatmadari, A. Ghodsi, A., Saif Naraghi, M., Ghanbari, N. The role of educational games on learning education concepts and comparing mathematical numbers of female elementary school students in Rey. Journal of Behavioral Sciences, 2011; 3(9): 85-100.
2. Soltani Dezky, S. A study on errors of fifth-grade female students in mathematical word problem solving based on Newman error analysis model. Unpublished MSc thesis in education planning, Payam-e Noor University, Faculty of Humanities, Education Dept, 2014.
3. Rastizadeh, E., Behzadi, M. H. The effect of teaching mathematical word problem solving on students’ error reduction based on Newman model. Fourteenth Conference of Iranian Mathematics Education, Shiraz, 2016.
4. Ahmad, A., Tarmizi, R. A., Nawawi, M. Visual Representations in Mathematical Word Problem Solving among Form Four Students in Malacca. Procedia-Social and Behavioral Sciences, 2010; 8, 356-361.
5. NCTM. Principles and standards for school mathematics. Reston, VA: NCTM, 2000.
6. Yusof, J., Lusin, S. The Role of Manipulatives in Enhancing Pupils’ Understanding on Fraction Concepts. International Journal for Infonomics (IJI), 2013; 6(3/4).
7. Post, T. R. Fractions: Results and Implications from National Assessment. The Arithmetic Teacher, 1981; 28(9), 26-31.
8. Trespalacios, J. H. The Effects of Two Generative Activities on Learner Comprehension of Part-Whole Meaning of Rational Numbers Using Virtual Manipulatives. Journal of Computers in Mathematics and Science Teaching, 2008; 29(3): 327-346.
9. Moradi Toopghara; L., Bayat, M. The role of manipulatives in trigonometry teaching. Roshd Mathematics Education, 2011; 28(4).
10. Amani Tehrani, M.Spectral not polar view on teaching and learning experimental sciences based on a new science education design", Special Issue on Science Education, 2005: 29.
11. Valizadeh, Z. The impact of mathematical structures on spatial geometry teaching in second and third high school grade of mathematics on the academic achievement of female students in District 5 of Tabriz in the academic year2014-2015. MSc thesis in Education Planning, Islamic Azad University, Marand Branch, 2014.
12. Kilpatrick, L. A., Suyenobu, B. Y., Smith, S. R., Bueller, J. A., Goodman, T., Creswell, J. D., et al. Impact of mindfulness-based stress reduction training on intrinsic brain connectivity. Neuroimage 56, 290–298. doi: 10.1016/j.neuroimage. 2011.02.03.
13. Haghverdi, M. Characteristics of mathematical word problems in guidance schools and facilitating ways of their solving process, Journal of Theory and Practice in Curricula, 2014; 2(3): 25-46.
14. Yeatts, K., Manipulatives: Motivating Mathematics, Dade Public Education Fund, Miami, FL, 1991.
15. Boggan, M., Harper, S., Whitmire, A. Using Manipulatives to Teach Elementary Mathematics. Mississipi State University. Journal of Instructional Pedagogies, 2009.
16. Smith, S. S. Early Childhood Mathematics (4th. ed). Boston: Pearson Education Using Manipulatives, 2009.
17. Hiebert, J. Making sense: teaching and learning mathematics with understanding, Heinemann, 1997.
18. Kaput, J. Toward a theory of symbol use in mathematics. In C. Janvier (Ed.), Problems of representation in mathematics learning and problem solving (pp. 159–195). Hillsdale, NJ: Lawrence Erlbaum Associates, 1987.
19. Lesh, R. Mathematical learning disabilities: considerations for identification, diagnosis and remediation. In R. Lesh, D.Mierkiewicz, & M.G. Kantowski (Eds), Applied Mathematical Problem Solving. Ohio: ERIC/SMEAC, 1979.
20. Moyer, P. S., Bolyard, J. J., Spikell, M. A. What Are Virtual Manipulatives? Teaching Children Mathematics, 2002; 8, 372-377.
21. Resnick, L. B., Omanson, S. F. Learning to understand arithmetic. In R. Glaser (Ed.), Advances in Instructional Psychology, 1987; 3: 41-95. Hillsdale, NJ: Erlbaum
22. Bakhshalizadeh, Sh. Identification of common misunderstandings in fourth-grade elementary students in mathematical context, Educational Research and Planning Organization, Education Studies Research Institute, 2013.
23. Leslie White, A. Numeracy, Literacy and Newman’s Error Analysis.University of Western Sydney, Allan Leslie White Journal of Science and Mathematics Education in Southeast Asia, 2010; 33 (2): 129 – 148.
24. Bautista, D. Mitchelmore, M., Mulligan, J. Factors influencing Filipino children’s solutions to addition and subtraction word problems. Educational Psychology, 2009; 29(6), 729-745.
25. Hj Raduan, I. Error analysis and the corresponding cognitive activities committed by year five primary students in solving mathematical word problems. Procedia Social and Behavioral Sciences, 2010; 2: 3836–3838.
26. Haser, Ç., Ubuz, B. Student’s conception of fractions: a study of 5th grade students. Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 2003; 24 : 64-69
27. Doosti, M., Reihani, I., Bakhshalizadeh, Sh. Students' problems in solving fraction word problems. Sixth National Conference on Education, 2014.

28. Aksu, M. Student performance in dealing with fractions. The Journal of Educational Research. 1997; 90(6): 375-380.
29. Doosti, M., Reihani, I., Bakhshalizadeh, Sh. Basic models and frameworks for understanding fractions in mathematical education. Third Virtual Conference on Education Leaders, 2014.
30. Gholami, A. Challenges of fraction education in Grade 6 primary school with cognitive approach and NCTM standard. Unpublished MSc Thesis, Shahid Beheshti University, Faculty of Mathematical Sciences, 2013.
31. Prakitipong, N., Nakamura, S. Analysis of mathematics performance of Grade Five students in Thailand Using Newman procedure, 2006. Retrieved August 14th, 2008 form home. Hiro shima- u. ac.jp/ciice/9-1 prakitipongnakamura.pdf
32. Sepeng, P., Sigola, S. Making Sense of Errors Made by leamers in Mathematical word problem solving. Mediterranean Jornal of Social Sciences. 2013; 4(13).
33. Vershaffel, L., Greer, B., Van Dooren, W. Words and Worlds. Netherlands: Sense Publishers, 2009.
34. Shin, T., Koehler, M.J., Mishra, P. Schmidt, D., Baran, E., Thompson, A. Changing technological pedagogical content knowledge (tpack) through course experiences. Paper presented at the International Conference of the Society for the Information and Technology & Teacher Education, 2009.
35. Acosta-Tello, A., Making Mathematics Word Problems Reliable Measures of student Mathematics Abilities. Jornal of Mathematics Education, 2010; 3(1), 15-26.
36. Tobias, J. M. Preservice elementary teachers' developing of rational number understanding through the social prespective and relationship among social and individual environment. A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Teaching and Learning Principles in the College of Education at the University of Central Florida Orlando, Florida, 2008.
37. Khodayari, M. Learning challenges in second-grade guidance school students in fraction problem solving. Unpublished MSc Thesis, Shahid Beheshti University, Faculty of Mathematical Sciences, 2013.
38. Clements. R. An Ivestigatin of the Status of Outdoor Play. Contemporary Issues in Early childhood, 2004; 5(1).