Original Article

The Effects of Visual Representations and Manipulatives on Reduction of Algebraic Misconceptions of Ninth-Grade Students

Vahid Alamian [1]*, Abdollah Barati [2] , Molouk Habibi [3]

[1] Faculty member Department of Mathmatics Farhangian University, Tehran, Iran.[2] Department of Mathmatics Farhangian University, Tehran, Iran [3] MA, Department of Mathmatics Farhangian University, Tehran, Iran.

Abstract

Representations have been identified as a factor affecting the development of mathematics by many researchers. Therefore, this study mainly aimed to investigate the effect of using visual representations and manipulatives on the reduction of algebraic misconceptions in ninth-grade students. The statistical population consisted of 650 male ninth-grade students in a high school of Ramhormoz in the academic year 2018-19. Male students (n = 87) in three ninth grade classes were selected from a single school. After a pretest, students' misconceptions were categorized into four categories of reading an inappropriate schema, new learning intervention in the previous schema, overgeneralization, and similarity of a mathematics word to a spoken word. The validity and reliability of data were confirmed by a researcher-made mathematics test. Two classes were then taught through teaching multiple strategies, visual, and manipulation, and one class via the traditional method. According to the results of qualitative and quantitative analysis, the use of multiple representations could affect the reduction of algebraic misconceptions of ninth grade students. Moreover, each of the visual representations and tools had an effect on the reduction of algebraic misconceptions in ninth-grade students. Among these representations, the mean posttest of the test group was higher than the subgroups of visual representations and manipulation. Mean scores indicated a significant effect after using visual representations and then manipulative representations.

Keywords: Multiple representations, Visual representations, Manipulative representations, Misconception, Algebra

Introduction

Given the role of mathematics in all scientific fields and the necessity of its teaching/learning, it is inevitable to use effective teaching methods that can interest students in mathematics and facilitate their learning; however, there are many obstacles that overturn achieving the goal, one of which is students' conceptual mistakes that are not the result of carelessness and non-concentration but are systematic errors in mathematics and do not occur accidentally. Students' misconception cases are often encountered in teaching experience. As such, students unexpectedly use subjects in answering questions or solving problems, or have thoughts that they think to be correct but are not compatible with mathematical logic. Students use the subject in different situations and believe in the wrong concepts formed in their minds. To reduce these misconceptions, teaching mathematics should be improved in the classroom. One of these practices is the use of multiple representations in teaching. Relationships, principles, and mathematical ideas can be demonstrated in multiple representations, including visual representations (e.g., graphics, images, or diagrams). Students observe or draw a shape, diagram or image and use them to think and communicate with a mathematical concept intuitively. Each type of representation clearly expresses different meanings of mathematical concepts. The use of multiple representations in mathematical concepts or problems and the implications of using them for educational outcomes has developed remarkably in recent years, leading to the development of new methods in mathematical education, particularly at the elementary and secondary levels [1, 2].

Address for correspondence: Mr. Vahid Alamian, Faculty Member Department of Mathmatics Farhangian University, Tehran, Iran.
Email: Vahid_alamian@yahoo.com

How to cite this article: Alamian, V., Barati, A. Habibi,M.. The Effects of Visual Representations and Manipulatives on Reduction of Algebraic Misconceptions of Ninth-Grade Students. Arch Pharma Pract 2020;11(S4):49-58.

The subject of algebra in secondary school mathematics, particularly in the ninth grade, is prone to many misconceptions for students. Besides, the researcher, as a teacher in this grade for many years, has dealt with plenty of students' misconceptions in algebra. Therefore, this study seeks to investigate the impact of multiple representations in teaching on reduction of common misconceptions in this important mathematics subject.

Brenner (1973) distinguishes three different representational systems, *viz*. enactive, iconic, and symbolic, which are used to describe the cognitive development process [3]. Mathematics educators have proposed different models for application of various representations, generally called multiple representations, in teaching mathematical concepts and relationships. One of these models, proposed by Lesh, emphasizes that an in-depth understanding of mathematical ideas is reflected in five different modes and the ability of connecting these five modes. The model (real-world situations, manipulatives, images or graphs, verbal, and written) as multiple representations in Lesh's perspective is of interest in the literature and mathematics education (quoted by Goya and Emami, 2013) [4]. Accordingly, the visual representation and manipulation are discussed here.

Mathematics Misconceptions

Skemp (1976-2006) argues that relational mathematics to be more acceptable as it can be used for new tasks, is easier to remember over time, is an effective goal in itself, and its relational schemas enlarge mathematical growth [5]. While there are many reasons for instrumental teaching (e.g., time constraint, difficulty, evaluation), Skemp argues that instrumental understanding is not really mathematical, and that these two kinds of knowledge (instrumental and relational) are so different that one may think they are two different kinds of mathematics. If the distinction is accepted, the word mathematics is actually a false friend for many children, as they find to their cost.

Battle (2010) also states that the misconception arises because the student has not understood or misunderstood the subject. These errors do not result from carelessness or neglect of activity and have deeper roots. Students' misconceptions may stem from their previous experiences and knowledge in daily life and may be maintained seriously by students and delay their learning outcomes. If misconceptions are not resolved in early school years, they will lead to problems at higher education levels and in people's daily lives. Understanding the causes and origins of students' misconceptions in mathematics helps teachers use appropriate designs in the classroom to prevent these misconceptions and correct them in cases of observation (quoted by Pourazima et al., 2012) [6].

Internal and external representations

By differentiating external and internal representations, Capote (1999) used the term "fusion" to emphasize the actions surrounded by experiences with the internalization of external representations [7]. Internal representations are usually associated with mental images that people create in their minds. Pape and Tchoshanov (2001) describe mathematical representations as an intrinsic conception of mathematical ideas or cognitive schemas, which are made by learners to create internal mental networks or an internal representation system [8]. For math teachers, the issue is how to facilitate students' ability to recognize, create, interpret, communicate, and translate between alternative representational modes. External representations are usually related to "knowledge and structure in the environment, as physical symbols, objects, or dimensions". Goldin and Stetingold (2001) pointed out that an external representation is "usually a sign or a configuration of signs, characters (visual lines), or objects" and that an external representation can symbolize "something other than itself" [9]. Many external representations in mathematics are conventional, which are "identified, defined, and accepted objectively" [9]. In the relevant literature, different definitions can be found concerning the concept of "representation" Abram (2001), Disessa and Sherin, (2000), Seeger et al (1998) summarized some of these definitions in very general terms as follows [10-12]: "... representation means any kind of mental state with particular content, mental reproduction of a previous mental state, an image, symbol, or sign, a symbolic tool for language learning, and one thing "rather than" something else ". Hall (1996) clarifies Dewey's view of representation as "representation is the transformation process of a problem state through inquiry and development of an experience during that activity [13]."

Representations can be categorized into two internal and external groups. Internal representations are defined as "separate cognitive modules inferred from human behavior that describe some aspects of mathematical thinking and problem solving process"; external representations, on the other hand, can be defined as "structured physical conditions that can be seen as the embodiment of mathematical ideas" [14].

According to the structuralist perspective, internal representations are located within the students' minds, and external representations are situated around students [15]. Goldin (1998) further researched these two phenomena and noted that any physical state, including mathematical ones, can be defined as external representations. For example, a line segment that illustrates the relationships between numbers, or a computer-based environment where the mathematical structure can be manipulated as external representations. On the other hand, the internal representation is only relatively universal meaning that what students conceptualize in their minds can be read as an internal representation [14]. The researcher agrees with the definitions of Lesh et al. (1987) [16]. External representations can be defined as the physical embodiment of ideas, concepts, and methods, by which learners can manipulate mathematical ideas. Edwards (1998) made a powerful and unique distinction between internal and external representations [17]. He explained that internal representations are something made only by learners, which can be figures, problem solving, or schemas. In contrast to internal representations, external representations are shared conventionally and have some common language to communicate with people. In mathematics, for example, tables, figures, and tree diagrams can be represented as external representations.

Categorization of representations

Goldin and Stiengold (2001) state that the interaction between internal and external representation systems is essential for teaching and learning mathematics. Seeger (1998) points out that the process of representation or demonstration involves identification, selection, and presentation of the concept through a method that is structurally similar and more understandable. Alternative methods of representing problems or algebraic concepts include shapes, diagrams, images, analogies, symbols, and signs (language and icons), while the use of representation is necessary for the student's understanding of mathematics concepts and the relationships among them. It is worth noting that each mode of representation provides only partial information and "emphasizes some aspects and hides others" [18], thus it is limited to some specific but important methods. Brenner (1973) distinguishes three different representational systems, *viz*. enactive, iconic, and symbolic, which are used to describe the cognitive development process. Vergnaud (1997) pointed out representation as a feature of mathematical concepts defined by three variables [19]: conditions that make the concept useful and meaningful, operations that can be applied to address this situation, and a set of symbolic, linguistic, and graphic representations that can be used to represent conditions and methods. Pirie (1998) associates representation with mathematical language, which is categorized as ordinary language, mathematical verbal language, symbolic language, visual representations, unspoken but shared assumptions, and quasi-mathematical language [20]. He states that the function of any representation is the transfer of mathematical ideas, and that any representation adds to this communication and helps to convey different meanings of a single mathematical concept.

Multiple representations in algebra

It is widely accepted that students need to use multiple representations. When traditional mathematics instruction and curriculum are examined, they only focus on symbol manipulation skills and rote memorization. Basic mathematics concepts are generally presented to students in abstract forms, particularly in algebra lessons, focusing only on the symbolic part of algebraic concepts [21]. In addition, instruction focuses mostly on the procedural skills, such as solving linear equations or finding the solution set of a given inequality system. Many math teachers avoid using multiple representations in algebra classes. However, algebra has utilized various representational systems to express ideas and processes and forms one of the cornerstones of school mathematics [22]. The importance of algebra was also supported by NCTM standards. According to NCTM: "to think algebraically, one must be able to understand patterns, relations, and functions, represent and analyze mathematical situations and structures using algebraic symbols, use mathematical models to represent and understand quantitative relationships, and analyze changes in various contexts. Each of these components evolves as students grow and mature" [23]. With the knowledge of this problem in algebra classes, students need to learn a variety of representations and their translations, and teachers need to introduce the concept of multiple representations to their students. LMRTM and Janvier's Representational Translations Model (JRTM) were proposed to provide better mathematical instruction and algebraic concept understanding in algebra classes. Lesh defined representation as an external (and therefore observable) embodiment of students' conceptualization of internal representations [16].

Background research

Ebrahimi (2016) studied the role of teachers in correcting students' mathematical misconceptions and reported that misconceptions resulted from by inappropriate teaching, informal thinking, or poor remembrance. Therefore, students' correct awareness of their mistakes and misconceptions in learning and solving mathematical problems is a decisive factor in the growth and development of their mathematical performance. This, therefore, necessitates that mathematics teachers to adopt appropriate practices and engage pupils in the lesson activities to enable them make necessary decisions with confidence about the correctness or incorrectness of their answers.

In a review of research on misconceptions in mathematical education, Ay (2017) examined 21 articles published during 2004-2015, and concluded that the number of studies on mathematical misconceptions had increased over the past five years [24]. Besides, most studies conducted in primary and secondary schools aimed at identifying misconceptions rather than eliminating them.

Kusumaningsih et al. (2018) conducted a study on the growth of algebraic thinking using multiple representation strategies in mathematics education" and detected an association between multiple representational strategies using a realistic approach and algebraic thinking ability [25]. Students with multiple strategies had better algebraic thinking ability than others.

Methods

This is an applied study regarding the objectives and semi-experimental in terms of data collection in which the dependent and independent variables are "algebraic misconceptions" and "multiple representations"; respectively.

Given the study topic, the statistical population consisted of all male students of the ninth-grade guidance high schools in Ramhormoz city, who were studying in the academic year 2017-18, with a total number of 650 ninth-grade male students. The statistical sample of this study consisted of 87 male students[1] in three ninth-grade classes from a single school, two of which were trained by multiple strategies and using them in problem solving, and one class by traditional method. It was attempted to answer each question through the field method by implementing the teaching approach based on the training of multiple representations.

Mathematics achievement test questions (pretest and posttest each containing 10 questions with a total score of 20) were designed by the researcher based on subject knowledge and skills, expert comments, and students' common misconceptions in the ninth-grade algebra- Chapter 5 (drawing on research findings that identified students' misconceptions and the author's own teaching experiences). Based on the results, necessary comparisons were made between the two teaching modes of teaching and answering research questions. Based on the number of experts who evaluated the questions, minimum acceptable CVR values should be based on Table 1.

Table 1: Minimum acceptable CVR values based on the number of experts

No. of experts	CVR	No. of experts	CVR	No. of experts	CVR
5	0·99	11	0·59	25	0·37
6	0·99	12	0·56	30	0·33
7	0·99	13	0·54	35	0·31
8	0·75	14	0·51	40	0·29
9	0·78	15	0·49		
10	0·62	20	0·42		

Results obtained from a survey of experts indicated good content validity of mathematics tests (approved by eight experts), which was confirmed with a value of approximately 75%. In this method, the test is performed once on a group and then the test is divided into two halves. The best way to split the test is to assign odd and even questions to separate groups. The correlation coefficient of the two half-test scores will be the reliability coefficient for each half-test. To this end, statistical software was used to measure Guttman split-half coefficient for mathematics tests, and values of 0.73 and 0.74 were obtained for mathematics tests (pretests), respectively, which are of acceptable reliability as being higher than 0.7.

The research design is semi-experimental of an unequal control group. In the test group, the design was implemented by teaching based on visual representations of manipulatives for topics of algebraic expressions, factorization, parsing, equation, and so on in the ninth grade. Prior to the implementation of this teaching plan, a test based on common and usual misconceptions was first conducted among the students and then the test group was taught based on the above-mentioned multiple representations according to the categories of misconceptions. In a control group, traditional teaching was performed simultaneously for the categories of misconceptions.

At the first stage of the research, the researcher classified students' misconceptions in answering pretest questions to implement the pretest design. In the academic year 2018-19, a pretest was taken after a preparatory instruction of the fifth chapter of the ninth grade mathematics textbook. The following misconceptions were mainly mentioned in this category:

- Reading an inappropriate schema
- New learning intervention in previous schema
- Overgeneralization
- Similarity of a mathematics word to a spoken word

Test group: Teaching visual representation and manipulatives according to students' classification into two subgroups

In visual representation of the experimental group, it was attempted to describe dynamically the teaching concepts, such as animations, among students as expressed in the appendix. In this situation, the researcher as a facilitator attempts to visualize algebraic abstract concepts among students to discuss algebraic concepts in terms of visualization and subjective embodiment.

In the representation of manipulatives, more emphasis is placed on students' manipulatives with objects in solving algebraic concepts. In this kind of representation teaching, students have to explain algebraic concepts in different ways according to their mental innovations through instrumental methods. Some students produced and designed algebraic concepts with physical tools in a non-abstract, real-world manner.

In the control group, 26 students with misconceptions were taught conventionally. The control group was taught in accordance with the topics approved by the Ministry of Education based on the chapter to be taught in due time. Based on this type of classification, the taught algebraic chapter contained the subjects of 1) algebraic expressions and the concept of factorization, and 2) parsing.

The plan was implemented for three months, and then, a posttest was taken to identify students' performance from the topics taught (both representational and conventional) to examine the reduction of misconceptions in the three groups.

[1] These students were selected from 102 students after taking a pretest and observing misconceptions. Of the 102 students, 87 subjects had one or more types of misconceptions.

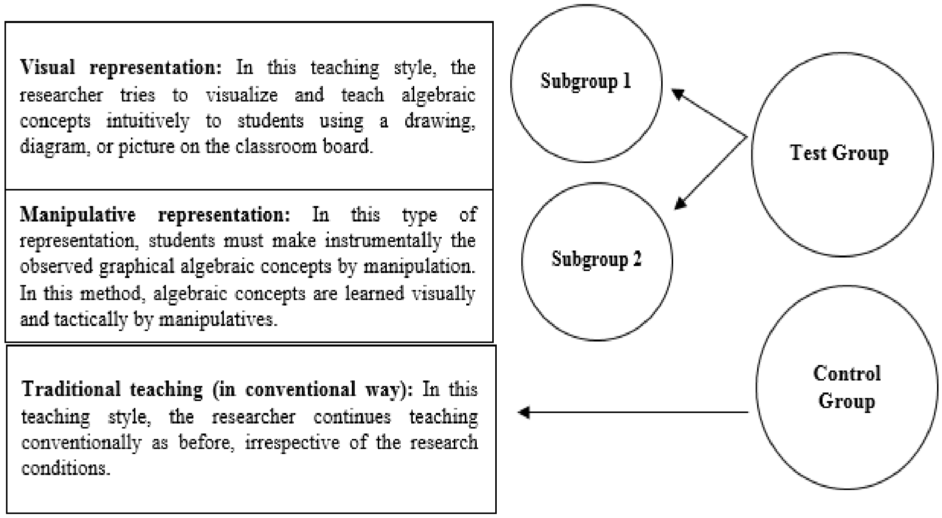

Chart 1: Research implementation plan based on the Lesh model

Findings

Pretest qualitative analysis - Determining misconceptions

According to the research questions, the researcher analyzes the findings in both qualitative and quantitative sections and examines samples of students' handwriting. It should be noted that more than one type of misconception may be observed in each of pretest questions. The responses of 87 out of 102 students with misconceptions were examined and summarized after the pretest. The students' misconceptions in 10 questions were divided into four main categories, some of which can be found in the following tables.

Table 2. Misconceptions of Question 5 in the pretest

Misconception types	Overgeneralization		Similarity of a math word to a spoken word	
Frequency / Question	Frequency	Percentage	Frequency	Percentage
Question 5	42	48.27	33	37.93

As shown in Table 2, two types of misconceptions were identified in Question 5, with overgeneralization (48.27%) being more than the similarity to a mathematical word.

Table 3. Misconceptions of Question 10 in the pretest

Misconception type	Reading an inappropriate schema	
Frequency / Question	Frequency	Percentage
Question 10	50	58.47

According to Table 3, a misconception of reading an inappropriate schema (58.47%) was identified in Question 10.

Posttest qualitative analysis - Determining misconceptions

Table 4. Misconceptions of Question 5 in posttest

Misconception types	New learning intervention in previous schema		Overgeneralization		Similarity of a math word to a spoken word	
Frequency / Question	Frequency	Percentage	Frequency	Percentage	Frequency	Percentage
Question 5	7	11·47	3	3·44	1	1·63

Table 4 represents three types of misconceptions identified in Question 5, with new learning intervention in the previous schema (11.47%) being higher than the other two misconceptions.

Table 5: Misconceptions of Question 10 in the posttest

Misconception types / Frequency Question	Reading an inappropriate schema		New learning intervention in previous schema		Overgeneralization	
	Frequency	Percentage	Frequency	Percentage	Frequency	Percentage
Question 10	8	13·11	6	9·83	7	11·47

As shown in Table 5, three types of misconceptions were determined in Question 10, with reading an inappropriate schema (13.11%) being higher than the other two misconceptions.

Pre-intervention sample responses of students

Question 5) Find the value of *x*.

$$I)\ -\frac{x+1}{3}+2=2x+\frac{1}{5}$$

$$II)\ \frac{x+2}{x-7}=-\frac{2}{5}$$

Question 10) Solve the following equations.

$$I)\ (x-2)(x-1)=0$$

$$II)\ (x-4)(x+1)=5$$

$$III)\ 3(2x-7)=2$$

Handwriting (a & b)

Handwriting (c)

Figure 1 Misconception - Reading an inappropriate schema in the pretest

In Questions 5 and 10 of the pretest, a number of students interpreted the expression separation in an equation to be the corresponding number equal to zero, and presented two different responses as shown in the figure below. In the student's answer, both expressions in brackets were taken equal to 5 to solve the first-order equations. In another answer, 1 was assumedly multiplied by 5 to separate the two expressions into two numbers and solve the two parentheses as first-order equations (handwritings a and b). The same practice is repeated in Question 5 of the pretest, where the students have exactly equalized the numerator and the denominator expressions separately given the equal sign to solve it as two separate first-order equations, as shown in the figure, without paying attention to the properties of expressive expressions, etc. (Handwriting c).

Post-intervention sample responses of students

This section interprets and analyzes qualitatively the students' responses to the posttest, which were examined after training based on multiple representations in the test group. Given that most of the misconceptions were resolved after the new intervention, some were even observed in the minor form:

Question 5) Find the results of the following statements.

$$I)\ \left(\sqrt{20}-4\right)\left(\sqrt{20}+4\right)=?$$

$$II)\ \left(\sqrt{20+4}\right)\left(\sqrt{20+4}\right)=?$$

Figure 2. Misconception - Overgeneralization in the posttest

A kind of overgeneralization is observed in the fifth posttest question, occurred due to the observation of radicals by the student who generalized the multiplication of two brackets and the sum of the numbers below the radical to $\sqrt{20}-4$ and $\sqrt{20}+4$; both exponentiation and multiplication were generalized in this situation. In another part of the question, the student generalized the multiplication and exponentiation to $\sqrt{20}$.

Question 10) If $x-\frac{1}{x}=8$, find the product of $x^2+\frac{1}{x^2}=?$

Figure 3. Misunderstanding - Reading an inappropriate schema, overgeneralization, and new learning intervention in the posttest

As shown in the answer to Question 10, the student(s) immediately added 8 to the second power and generalized this power. After simplification and taking common denominator, the student confused the factoring with the schemas he had in mind before, concerning simplification of the common factor, thereby eliminating the common factor leading to some kind of intervention. After this operation, he set the left side to zero and solved the equation incompletely and mistakenly because the student had practiced with cases of first-order equations in order to eliminate misconception of reading inappropriate schema in the test group, therefore, he was not able to solve this problem as in previous exercises.

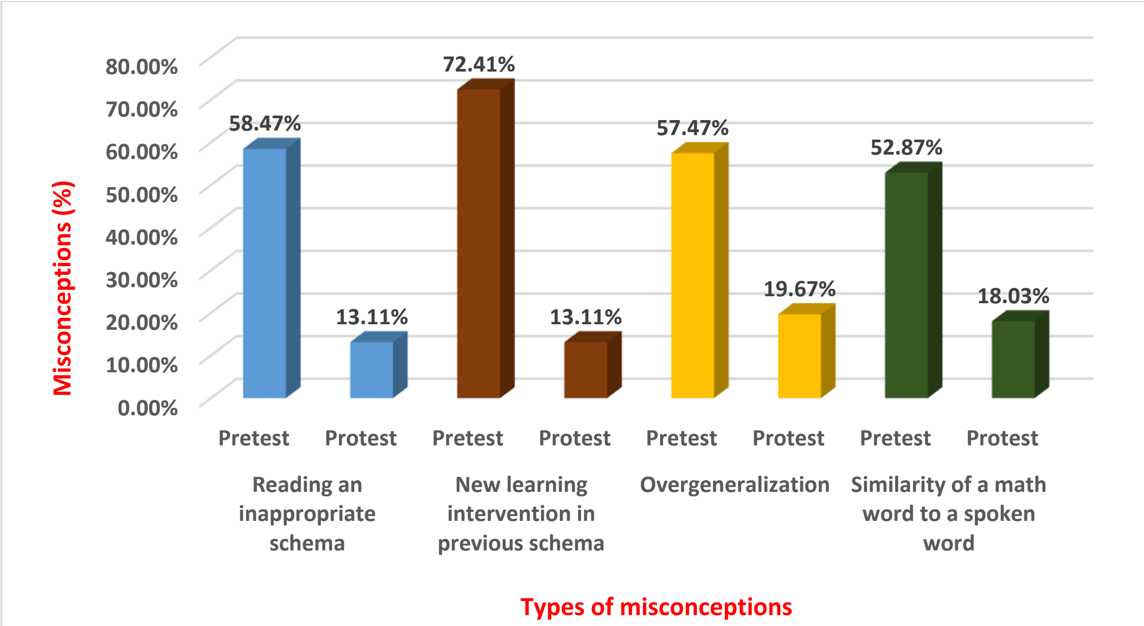

Figure 2 Comparisons between maximum and minimum percentages of misconceptions in pre- and posttest

As illustrated in Figure 2, the maximum level of misconceptions in the pretest decreased significantly after the new educational intervention based on multiple representations. Among the four categories of misconceptions, new learning intervention in the previous schema had significant changes compared with the other three types of misconceptions.

Quantitative analysis

The results of the descriptive statistics are described in this section. The central indices and distributions of data were evaluated using data collected by mathematics tests to examine the learning level of algebraic concepts.

Table 6: Results of descriptive statistics

Groups / Indices	Pretest – control group	Protest – control group	Pretest – test group	Protest – test group
Number	26	26	31	31
Mean	11·26	12·76	11·35	18·09
SD	1·66	1·25	1·78	1·32

In Table 6, significant differences are observed between the mean posttest scores of the control and test groups. The mean posttest score of the test group in terms of using visual representation and manipulation (18.09) is significantly higher than the control group (12.76). In addition, the highest and the lowest dispersions between the scores are observed in the pretest and the posttest of the test and control groups, respectively.

Table 7: Results of descriptive statistics after the educational intervention based on multiple representations

Groups / Indices	Visual representation	Manipulative representation
Number	17	14
Mean	18·47	17·64
SD	1·23	1·33

As shown in Table 7, the average scores are higher than those of manipulative representations after using visual representations with the lowest average. Furthermore, the highest and the lowest dispersions between scores are noticed in the use of manipulation and the use of visual representations, respectively.

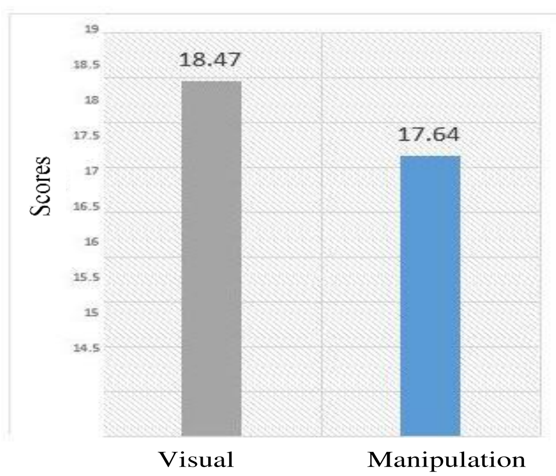

Figure 3 Mean scores of students based on using multiple representations

In Figure 3, there are significant differences between the mean scores after using multiple representations, with somewhat similar averages of manipulative and handwriting representations. Inferential statistics were used to clarify this issue.

Inferential statistics

Data normalization: In order to use and select appropriate statistical tests in the inferential statistics section, it is necessary to verify the normality of the test and questionnaire data. Therefore, the following hypotheses are the aims of this test:

H0: Data are normal.

Alternative (contrary) hypothesis: Data are not normal.
This hypothesis was tested using the Kolmogorov-Smirnov test.

Table 8. Normalization results

Indices / Groups	Pretest – control group	Protest – control group	Pretest – test group	Protest – test group
Test statistic	0·86	1·02	0·80	1·37
P-value	0·44	0·24	0·53	0·06

The P-values greater than 0.05 (Table 8) indicate that the data of scores are normal. Thus, the data were analyzed by the analysis of variance (ANOVA).

Analysis of findings

The ANOVA was used to compare the data and the presence or absence of significant differences given the normality of the scores for the ninth-grade algebraic concepts to more precisely identify the main hypothesis and examine significant differences between the three variables. Before doing one-way ANOVA, the hypotheses required for this test are examined as follows:

Hypothesis I: Independent random samples are taken from each population. The randomness of samples was examined using the runs test.

Table 9: Results of the runs test

Indices / Groups	Test statistic	P-value
Pretest - control	0·20	0·84
Pretest – test group	0·40	0·68
Protest - control	-0·17	0·86
Protest – test group	-0·31	0·75

A P-value greater than 0.05 (Table 9) reveals that the samples were taken at random.

Hypothesis II: The data for each group (corresponding to each community) are distributed normally, which was tested using the Kolmogorov-Smirnov test (Table 8), suggesting the normality of data. This hypothesis is not required since the number of samples is more than 15.

Therefore, the ANOVA test was used to examine significant differences between more than two populations. The purpose is to test the following hypotheses:

H0: Mean values of the control and test groups are the same in the pretests.

Alternative hypothesis: The mean values of the control and test groups are not the same in the pretests.

Table 10: ANOVA test results - pretests

Indices	Sum of squares	df	Mean square	F	P-value
Between groups	4·06	2	2·03	0·60	0·55
Within groups	283·67	84	3·37		
Total	287·74	86	---		

A P-value greater than 0.05 (0.55 > 0.05; Table 10) confirms the H_0. The means of the control and test group are the same in the pretests. The following hypothesis was tested after obtaining posttest scores in the groups based on representations:

H_0: Mean values of the control and test groups are the same in the posttests.

Alternative (Contrary) hypothesis: Mean values of the control and test groups are not the same in the posttests.

Table 11: ANOVA test results – posttests

Indices	Sum of squares	df	Mean square	F	P-value
Between groups	909·98	4	227·49	165·49	0·000
Within groups	112·72	82	1·37		
Total	1022·71	86			

A P-value less than 0.05 (0.000 < 0.05; Table 11) rejects the H_0, meaning that mean values of the control and test groups are not the same in the posttests.

Table 12: Results of equality of variances - posttests

Leven's statistic	df 1	df 2	P-value
1·41	4	82	0·23

A P-value greater than 0.05 (Table 12) suggests the equality of variances. Therefore, pairwise significant differences in the posttest scores of the groups were examined using Tukey's HSD test based on the representations.

Table 13: Results of Tukey's HSD test

Group	Groups	Mean difference	SD error	P-value	Confidence interval (95%)	
					Upper limit	Lower limit
Control	Visual representation	-7·89	0·36	0·000	-8·91	-6·87
	Manipulative representation	-7·06	0·38	0·000	-8·15	-5·98
Visual representation	**Control**	**7·89**	**0·36**	**0·000**	**6·87**	**8·91**
	Manipulative representation	0·82	0·42	0·29	-0·35	2·00
Manipulative representation	Control	7·06	0·38	0·000	5·98	8·15
	Visual representation	-0·82	0·42	0·29	-2·00	0·35

The results of Tukey's HSD test (Table 13) reveal significant differences between mean posttest scores of students in the control and test groups (0.000 < 0.05). It can, therefore, be concluded that the use of multiple representations has an effect on the reduction of algebraic misconceptions in ninth-grade students. Additionally, each of the visual representations and manipulatives had an impact on the reduction of algebraic misconceptions in the ninth-grade students.

Discussion and Conclusion

Students presented the best performance in learning algebraic concepts using visual representation among other representations in this study. Due to the use of intuitions and diverse forms, visual representation enables students frequently to visualize mentally. The use of manipulatives, with somewhat better effect on students' performance, was in the second place after the visual representation because of using the facilities, providing some equipment, and lack of some facilities for few students.

Answer to the first question

Does the use of visual representations have an impact on the reduction of algebraic misconceptions in ninth-grade students?

The quantitative and qualitative findings demonstrated that the use of visual representation had an impact on the reduction of algebraic misconceptions in ninth-grade students. Among multiple representations, visual representation exerted the uppermost effect on the reduction of misconceptions. In visual representations, the use of images and intuitions of students can enhance their visual ability to learn abstract algebraic concepts hence they can organize such concepts in their minds and thoughts. In visual representation, the student can have a visual view of algebraic concepts, and this kind of enablement, if developed, can be generalized to other algebraic topics. The creation and development of visual abilities for algebraic concepts can flourish other abilities.

Answer to the second question

Does the use of manipulative representations have an impact on the reduction of algebraic misconceptions in ninth-grade students?

The quantitative and qualitative findings revealed that the use of manipulative representations had an effect on the reduction of algebraic misconceptions in ninth-grade students. Among multiple representations, manipulation was found to have the highest effect on the reduction of misconceptions after visual representation. After the use of their self-made educational tools, students could better communicate with abstract concepts of algebra. This kind of manipulative communication can enhance simultaneously motor and mental abilities, both of which can influence the learning of mathematics concepts.

In fact, students should learn automatically what they hear and write. Accordingly, visual representation and manipulatives could influence better performance of students. In visual representation, visualization intuitions in the objective world could better help students learn about abstract mathematical concepts. The four categories of misconception occurred in this study mitigated significantly by the pre-designed, organized representation-based teaching, which is somewhat in line with those of Ahmadi and Yaftian (2017), Ebrahimi (2016) [26], Nielsen and Bustick (2018), Kusumaningsih et al. (2018), Ravo and Matthews (2017), and others.

Recommendations

- Based on the better results of visual representation and manipulatives, expert professionals can design and deliver instructional content based on these two representations for concepts based on the validity of such representations.
- Based on the reports of this study, it is recommended to design and execute online or software training practices for each of the representations under the supervision of relevant experts to provide the online usability of the representations.
- Based on the results of students' performance in this study, the most practical representation can be evaluated in students' daily lives through teaching strategies of teachers for algebraic concepts to introduce the most practical representation.

References

1. Panasuk RM. Three Phase Ranking Framework for Assessing Conceptual Understanding in Algebra Using Multiple Representations. Education. 2010 Dec 1;131(2).
2. Panasuk RM. Taxonomy for assessing conceptual understanding in algebra using multiple representations. College Student Journal. 2011 Jun 1;45(2):219-33.
3. Bruner, J. Beyond the information given. New York: Norton & Company, 1973.

4. Gooya, Z., Emami, A. Representations and their Roles in Understanding the Concept of Function, Roshd Journal of Mathematics Education, 2013; 114, 24-35.
5. Skemp, R. R. Relational understanding and instrumental understanding. Mathematics Teaching in the Middle School, (1976/2006); 12(2),88-95.
6. Pourazima, Z, Reihani, A, Bakhshali Zadeh, Sh. How do students think about decimal numbers? Proceedings of the Fourth National Conference on Education, Shahid Rajaee Teacher Training University, Tehran, 2012.
7. Kaput, J. Linking representations in the symbol systems of algebra. In S. Wagner & C. Kieran (Eds.), Research issues in the teaching and learning of algebra (pp. 167-194). Reston, VA: National Council of Teachers of Mathematics, 1999.
8. Pape, S. J., Tchoshanov, M. A. The role of representation(s) in developing mathematical understanding. Theory into Practice, 2001; 40(2), 118-125.
9. Goldin, G., Shteingold, N. System of mathematical representations and development of mathematical concepts. In F. R. Curcio (Ed.), The roles of representation in school mathematics (pp. 1-23). Reston, VA: National Council of Teachers of Mathematics, 2001.
10. Abram, J. P. Teaching mathematical modeling and the skills of representation. In A. A. Cuoco, & F. R. Curcio (Eds.), The Roles of Representation in School Mathematics (pp. 269-282). Reston: NCTM, 2001.
11. diSessa, A. A., Sherin, B. Meta-representation: An introduction. Journal of Mathematical Behavior, 2000; 19(4): 385-398.
12. Seeger, F., Voight, I., Werschescio, V. Representations in the mathematics classroom: reflections and constructions. In F. Seeger, I, 1998.
13. Hall, R. Representation as shared activity: situated cognition and Dewey's cartography of experience. The Journal of the Learning Sciences, 1996; 5(3), 209-238.
14. Goldin, G. A. Representational systems, learning, and problem solving in mathematics. Journal of Mathematical Behavior, 1998; 17 (2): 137-165.
15. Cobb, P., Yackel, E., Wood, T. A constructivist alternative to the representational view of mind in mathematics education. Journal for Research in Mathematics Education, 1992; 23(1): 2-33.
16. Lesh, R., Post, T., Behr, M. Dienes revisited: multiple embodiments in computer environment. In I. Wirsup, & R. Streit (Eds.), Development in School Mathematics Education around the World (pp.647-680). Reston, VA: National Council of Teachers of Mathematics, 1987.
17. Edwards, L. D. Embodying mathematics and science: Microworld as representations. The Journal of Mathematical Behavior, 1998; 17(1), 53-78.
18. Dreyfus, T., Eisenberg, T. Intuitive functional concepts: A baseline study on intuitions. Journal for Research in Mathematics Education, 1982; 13(5), 360-380.
19. Vergnaud, G. The nature of mathematical concepts. In P. Bryant (Ed.), Learning and teaching mathematics (pp. 5-28). East Sussex: Psychology Press, 1997.
20. Pirie, S. E. B. Crossing the gulf between thought and symbol: Language as stepping-stones. In H. Steinbring, M., G. B. Bussi & A. Sierpinska (Eds.), Language and communication in the mathematics classroom (pp. 7-29), 1998.
21. MEB. İlköğretim Okulu Ders Programları: Matematik Programı 6-7-8. İstanbul: Milli Eğitim Basımevi, 2002.
22. Lubinski, C. A., Otto, A. D. Meaningful mathematical representations and early algebraic reasoning. Teaching Children Mathematics, 2002; 9(2), 76-91
23. National Council of Teachers of Mathematics. Principles and standards for school mathematics. Reston, VA: Author, 2000.
24. Ay, Y. A review of research on the misconceptions in mathematics education, Education Research Highlights in Mathematics, Science and Technology, 2017; 21- 31.
25. Kusumaningsih W. Improvement Algebraic Thinking Ability Using Multiple Representation Strategy on Realistic Mathematics Education. Journal on Mathematics Education. 2018 Jul;9(2):281-90.
26. Ibrahimi, NA. The role of teachers in correcting students' mathematical misconceptions. 14th Iranian Mathematics Education Conference, Shiraz, Iran, 2012.